

EXAMINATION #1 VERSION A
“Mathematical Tools”
September 7, 2021

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [2 pts each—30 pts total]

(1) A function $y = f(x)$ slopes upward if its derivative equals

- a. zero.
- b. infinity.
- c. 2.
- d. -1.
- e. cannot be determined from the information given.

(2) Suppose $y = 5 + (2/x)$. Then the derivative of y with respect to x is given by the formula

- a. $dy/dx = 2$.
- b. $dy/dx = -2/x^2$.
- c. $dy/dx = 2/x^2$.
- d. $dy/dx = 5x$.
- e. none of the above.

(3) Suppose $y = 3x^2 + 5x + 7$. Then the derivative of y with respect to x is

- a. $dy/dx = 6$.
- b. $dy/dx = 5$.
- c. $dy/dx = 6x + 5$.
- d. $dy/dx = 3x + 5$.
- e. $dy/dx = 5x + 7$.
- f. $dy/dx = 6x^2 + 6x + 7$.

(4) Suppose $y = (2+x)^2$. Then the derivative of y with respect to x is given by

- a. $dy/dx = 2$.
- b. $dy/dx = 2x$.
- c. $dy/dx = 2(2+x)$.
- d. $dy/dx = (2 + 2x)$.
- e. none of the above.

(5) Suppose $y = x^{1/3}$. Then the derivative of y with respect to x is given by

- a. $dy/dx = x/3$.
- b. $dy/dx = x^{-2/3}/3$.
- c. $dy/dx = x^{4/3}$.
- d. $dy/dx = (-2/3)x^3$.
- e. none of the above.

(6) Suppose $y = 2(4x+5)^3$. Then the derivative of y with respect to x is

- a. $dy/dx = 2$.
- b. $dy/dx = (4x+5)^2$.
- c. $dy/dx = 6(4x+5)^2$.
- d. $dy/dx = 24(4x+5)^2$.
- e. $dy/dx = 2(4x+5)^2$.

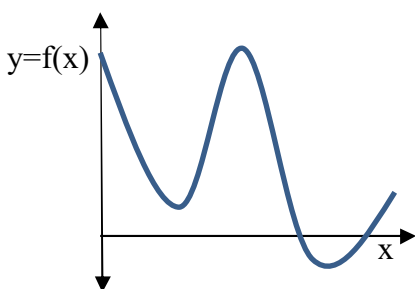
(7) Which of the following functions has constant slope (or derivative)?

- a. $y = 5 + 4x$.
- b. $y = 5 + (3/x)$.
- c. $y = 5 + 5x + (3/4)x^4$.
- d. $y = 5x^{-4}$.
- e. $y = \ln(4x)$.
- f. $y = \exp(3x)$.

(8) If x increases by 4 percent, then $\ln(x)$ increases by about

- a. $\ln(4)$, or about 1.386 units.
- b. 0.04 percent.
- c. 0.04 units.
- d. 4 percent.
- e. 4 units.

The next question refers to the following graph of $y = f(x)$.



(9) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at

- a. no point on the graph.
- b. one point on the graph.
- c. two points on the graph.
- d. three points on the graph.
- e. four points on the graph.
- f. more than four points on the graph.

(10) Suppose we have a function $y = f(x)$, which is continuously differentiable. At this function's maximum value,

- a. $f(x) = 0$.
- b. $df/dx = 0$.
- c. $f(x) = 1$.
- d. $df/dx = 1$.
- e. df/dx is as large as possible.

(11) If y is proportional to x , (that is, if $y = ax$, where a is an unknown constant) then the elasticity of y with respect to x equals

- a. x .
- b. a .
- c. zero.
- d. one.
- e. cannot be determined from information given.

(12) Consider the following functions. Which has constant elasticity?

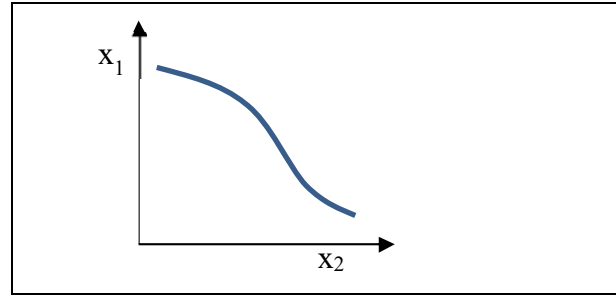
- a. $y = 5 + 4x$.
- b. $y = 5 + (3/x)$.
- c. $y = 5 + 5x + (3/4)x^4$.
- d. $y = 5x^{-4}$.
- e. $y = \ln(4x)$.
- f. $y = \exp(3x)$.

(13) A straight line has constant

- a. slope.
- b. elasticity.
- c. both of the above.
- d. none of the above.

(14) Suppose y depends on both x_1 and x_2 , so that $y = f(x_1, x_2)$. By definition, $\partial y / \partial x_1$, the partial derivative of y with respect to x_1 , is the ratio of the change in y to the change in x_1 when

- a. x_1 is held constant.
- b. x_2 is held constant.
- c. y is held constant.
- d. x_1 is held equal to x_2 .



(15) By definition, all points along the curve in this graph have identical values of

- a. x_1 .
- b. x_2 .
- c. y .
- d. the marginal rate of substitution.
- e. all of the above.

The next question refers to the following graph of a level curve, or contour, of the function $y = f(x_1, x_2)$.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals -4 at a particular value of x . Moreover, the elasticity of y with respect to x equals -0.5 . Further suppose that x increases by 3 units. [Hint: Some of this information is extraneous and not needed to answer this question.]

- a. Will y increase or decrease?
- b. By about how much?

units

(2) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 4$, and $\partial y / \partial x_2 = 5$, and that the partial elasticities are $\epsilon_1 = 0.2$ and $\epsilon_2 = 0.5$. Further suppose that x_1 increases by 5 percent and simultaneously x_2 increases by 6 percent. [Hint: Some of this information is extraneous and not needed to answer this question.]

- a. Will y increase or decrease?
- b. By about how much?

percent

(3) [4 pts] Total cost for a firm equals unit cost (also called average cost) times output quantity. Suppose unit cost decreases by 2 percent and output quantity increases by 10 percent.

a. Will total cost *increase* or *decrease*?

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b. By about how much?

percent

(4) [4 pts] The capital-labor ratio for an industry equals capital input divided by labor input. Suppose capital input increases by 5 percent and labor input increases by 2 percent.

a. Will the capital-labor ratio *increase* or *decrease*?

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b. By about how much?

percent

(5) [8 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 6$, and $\partial y / \partial x_2 = 3$. First, suppose that x_1 increases by 5 units but x_2 does not change.

a. Will y *increase* or *decrease*?

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b. By about how much?

units

Now suppose that x_1 increases by 5 units but we want y to remain constant. To keep y constant, we must change the value of x_2 .

c. Must x_2 *increase* or *decrease*?

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d. By about how much?

units

(6) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 4$, and $\partial y / \partial x_2 = 3$. Now consider a graph of the level curve of this function, with x_1 on the vertical axis and x_2 on the horizontal axis.

a. Does the level curve of the function slope *up* or *down* at that point?

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b. Give the slope of the level curve at this point.

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III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = -3x^2 + 18x + 5$.

- a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

- b. Compute the value x^* that maximizes this function.

- c. For what range of values of x does this function slope up? For what range of values does it slope down?

- d. Find the maximum value, y^* , of the function itself.

(2) [Partial elasticities: 6 pts] Suppose $y = x_1^3 (x_2 - 1)^2$.

- a. Find an expression for ε_1 , the partial elasticity of y with respect to x_1 . The variable y should *not* appear in your answer. Simplify if possible.

- b. Find an expression for ε_2 , the partial elasticity of y with respect to x_2 . The variable y should *not* appear in your answer. Simplify if possible.

(3) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = (x_1 - 3)^2 (x_2 - 5)^3$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the partial derivative of y with respect to x_1 .

- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

(4) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = 2x_1^{1/2} + 4x_2^{1/2}$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the partial derivative of y with respect to x_1 .

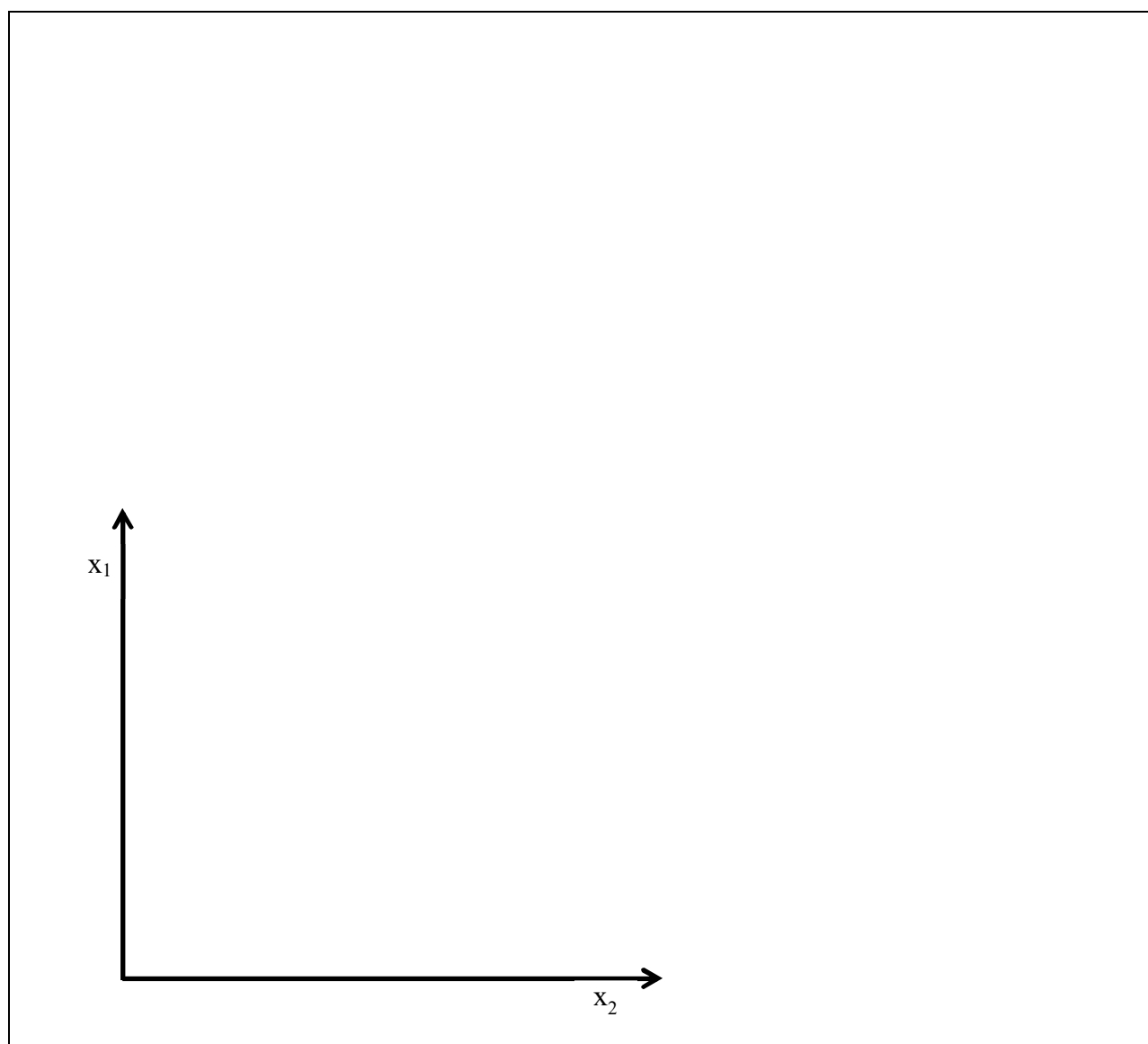
- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

IV. CRITICAL THINKING: [4 pts] Answer *one* question below (your choice). Circle the question you are answering. Justify your answer and show your work.

(1) Suppose $y = f(x_1, x_2)$. Further suppose $\partial y / \partial x_1 = \partial y / \partial x_2$ exactly for all values of x_1 and x_2 . Do the level curves slope up or down? Are they curved or straight? Justify your answers. Sketch a level curve of this function in the graph below.

(2) Suppose the quantity of output produced by a factory is a function of capital input and labor input. Let ϵ_K denote the partial elasticity of output with respect to capital input and let ϵ_L denote the partial elasticity of output with respect to labor input. Further suppose $\epsilon_K + \epsilon_L = 1$. If labor and capital both simultaneously increase by 5 percent, does output *increase* or *decrease*? By how much? Justify your answer. (No graph is needed.)



[end of exam]