

EXAMINATION #3 ANSWER KEY
“Firms and Competition”

Version A

I. Multiple choice

- (1)d. (2)b. (3)d. (4)e. (5)c. (6)b. (7)a. (8)b. (9)c. (10)d.
(11)a.

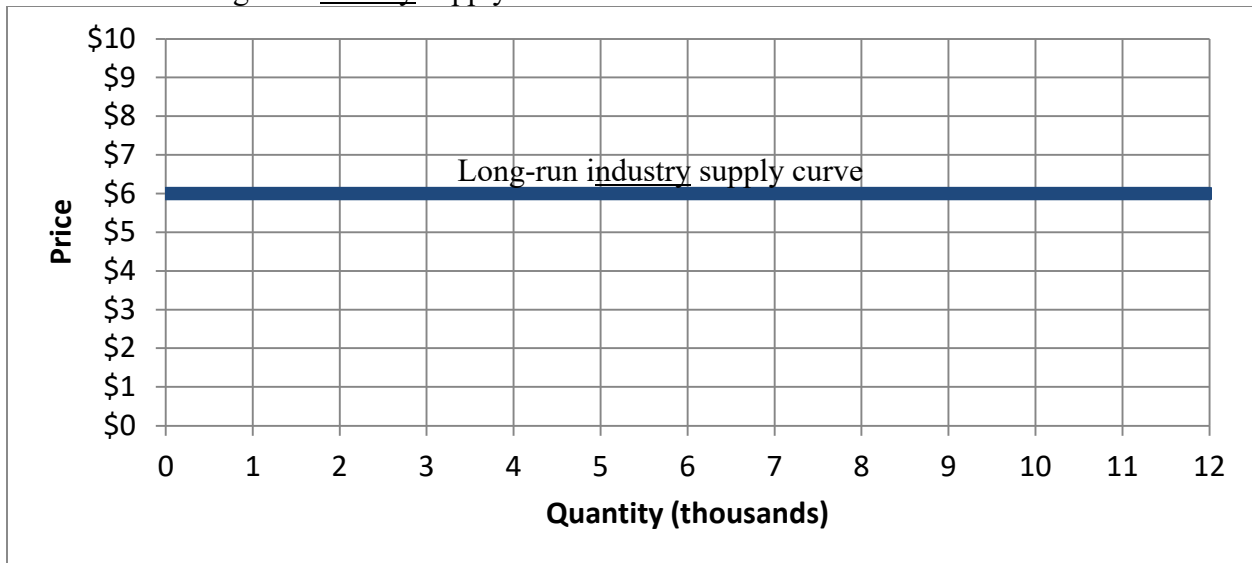
II. Short answer

- (1) a. 6 units. b. 2 units. c. \$90.
d. 7 units. e. \$135.
- (2) a. 0 thousand (because price is below shutdown price).
b. 13 thousand (using rule $P=MC$ to find q).
c. 11 thousand (using rule $P=MC$ to find q).
d. \$6 (because breakeven price = $\min(\text{SATC})$).
e. \$3 (because shutdown price = $\min(\text{SAVC})$).
- (3) a. \$5. b. 5 thousand. c. \$4 per pumpkin.
d. \$7 per pumpkin. e. decrease. f. \$6 thousand.
g. decrease. h. \$12 thousand. i. \$15 thousand.
j. \$3 thousand.

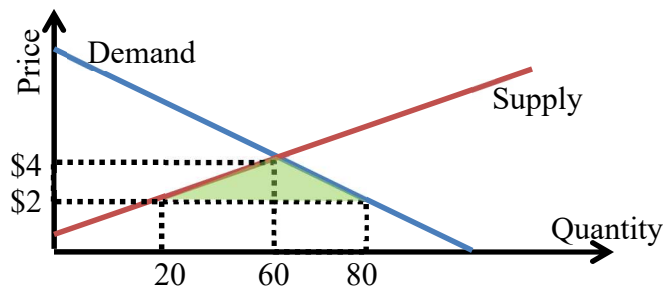
III. Problems

- (1) [Production functions]
a. $MP_1 = 3$. No, there are NO diminishing returns to input 1, because as x_1 increases (and x_2 is held constant), MP_1 remains constant.
b. $MRSP = MP_2/MP_1 = \frac{2}{3}$. No, this function does NOT have diminishing MRSP, because as x_1 decreases and x_2 increases, MRSP remains constant.
c. Check returns to scale:
 $f(ax_1, ax_2) = 3(ax_1) + 2(ax_2) - 5 > 3ax_1 + 2ax_2 - a5 = aq$, for all $a > 1$.
Thus, multiplying all inputs by the same factor (a) causes output to increase by a larger factor. So this production function has INCREASING returns to scale.
- (2) [Cost minimization]
a. Equation for isoquant: $60 = 4x_1^{1/2}x_2^{1/2}$ or $15 = x_1^{1/2}x_2^{1/2}$ or $225 = x_1x_2$.
b. $MRSP = MP_2/MP_1 = \frac{2x_1^{1/2}x_2^{-1/2}}{2x_1^{-1/2}x_2^{1/2}} = x_1/x_2$.
c. Set $MRSP = \$18/\2 and solve jointly with $60 = 4x_1^{1/2}x_2^{1/2}$, to get $x_1^*=45$ and $x_2^*=5$.
d. $TC(60) = 45 \times \$2 + 5 \times \$18 = \$180$.

- (3) [Long-run profit maximization and supply]
- $AC = TC/q = 0.01 q^2 - 0.4 q + 10$.
 Set $0 = dAC/dq = 0.02 q - 0.4$ and solve to get $q_{ES} = 20$.
 - Breakeven price = minimum $AC = AC(q_{ES}) = \$6$.
 - A supply curve shows how much will be produced for any given price. If $P >$ minimum average cost, the profit-maximizing firm will choose an output level where $P = MC(q)$, and if $P <$ minimum average cost, it will produce nothing. So the firm's supply curve is given by the following equations.
 If $P \geq \$6$, $P = MC(q) = dTC/dq = 0.03 q^2 - 0.8 q + 10$.
 If $P \leq \$6$, $q = 0$ (firm shuts down).
 - The long-run industry supply curve is a horizontal line at minimum AC :



- (4) [Welfare effects of international trade]
- Set $Q_D = Q_S$ and solve to get $P^* = \$4$ and $Q^* = 60$.
 - With international trade, $P_W = \$2$. Substituting into demand and supply gives $Q_D = 80$ and $Q_S = 20$, so the country **IMPORTS** $80 - 20 = 60$ units.



- Consumer surplus increases by \$140, the area of the large trapezoid.
- Producer surplus decreases by \$80, the area of the small trapezoid.
- The country as a whole gains $\$140 - \$80 = \$60$, the area of the green triangle.

IV. Critical thinking

(1) “Unfettered competition” does not destroy industries. Competition in the long run drives price to average cost, and thus drives *economic* profit to zero, but not *accounting* profit. Zero *economic* profit just means that firms are making a normal rate of return on capital, so firms remaining in the industry no longer have any incentive to leave, but typically those firms have positive *accounting* profits and they do not fail.

(2) Since the firm wants to produce 100 units of output, it must be on this target isoquant:

$$(1) \quad 100 = x_L^{0.5} x_K^{0.25} x_E^{0.25} .$$

Now the firm wants to produce those 100 units at least cost. Assuming that it has chosen the correct amount of x_L , then it must choose the other two inputs to minimize cost. Treating x_L as a constant, we therefore must have

$$(2) \quad \frac{MP_K}{MP_E} = \frac{\text{price of capital}}{\text{price of energy}} \quad \text{or} \quad \frac{x_E}{x_K} = \frac{4}{1} .$$

By similar reasoning, treating x_K as a constant, we must have

$$(3) \quad \frac{MP_L}{MP_E} = \frac{\text{price of labor}}{\text{price of energy}} \quad \text{or} \quad \frac{2 x_L}{x_K} = \frac{16}{1} .$$

Again by similar reasoning, treating x_E as a constant, we must have

$$(4) \quad \frac{MP_L}{MP_K} = \frac{\text{price of labor}}{\text{price of capital}} \quad \text{or} \quad \frac{2 x_L}{x_K} = \frac{16}{4} .$$

In fact, the fourth equation is redundant, since it equals the third equation divided by the second equation. Put differently, if the second and third equations hold, then the fourth equation holds automatically.

[The solution to these equations, which is not required for full credit, turns out to be $x_L^*=50$, $x_K^*=100$, and $x_E^*=400$.]

Version B

I. Multiple choice

(1)b. (2)d. (3)f. (4)f. (5)b. (6)a. (7)b. (8)c. (9)d. (10)c.
(11)b.

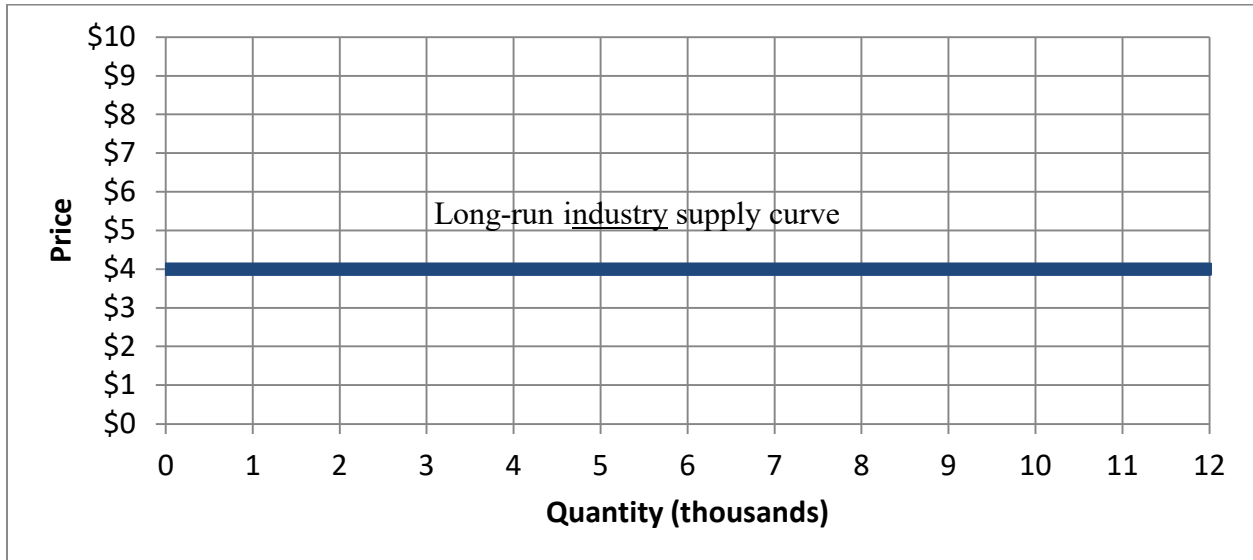
II. Short answer

(1) a. 2 units. b. 6 units. c. \$90.
d. 3 units. e. \$95.

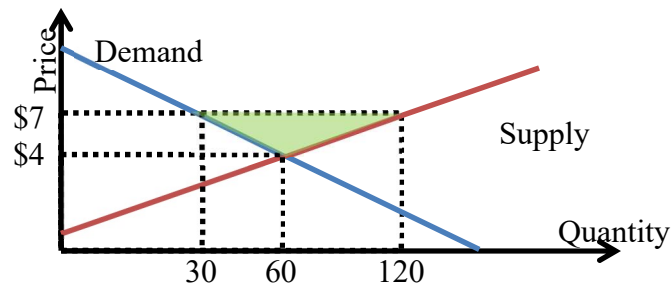
- (2) a. 0 thousand (because price is below shutdown price).
b. 11 thousand (using rule $P=MC$ to find q).
c. 12 thousand (using rule $P=MC$ to find q).
d. \$7 (because breakeven price = $\min(\text{SATC})$).
e. \$2 (because shutdown price = $\min(\text{SAVC})$).
- (3) a. \$5. b. 9 thousand. c. \$6 per pumpkin.
d. \$3 per pumpkin. e. increase. f. \$8 thousand.
g. increase. h. \$16 thousand. i. \$27 thousand.
j. \$3 thousand.

III. Problems

- (1) [Production functions]
a. $MP_1 = \frac{3}{2}x_1^{-1/2}$. YES, there are diminishing returns to input 1, because as x_1 increases (and x_2 is held constant), MP_1 decreases.
b. $MRSP = MP_2/MP_1 = \frac{2x_1^{1/2}}{3x_2^{1/2}}$. YES, this function has diminishing MRSP, because as x_1 decreases and x_2 increases, MRSP decreases.
c. Check returns to scale:
$$f(ax_1, ax_2) = 3(ax_1)^{1/2} + 2(ax_2)^{1/2} = 3a^{1/2}x_1^{1/2} + 2a^{1/2}x_2^{1/2}$$
$$= a^{1/2}(3x_1^{1/2} + 2x_2^{1/2}) = a^{1/2}q < aq, \text{ for all } a > 1.$$
Thus, multiplying all inputs by the same factor (a) causes output to increase by a smaller factor. So this production function has DECREASING returns to scale.
- (2) [Cost minimization]
a. Equation for isoquant: $80 = 4x_1^{1/2}x_2^{1/2}$ or $20 = x_1^{1/2}x_2^{1/2}$ or $400 = x_1x_2$.
b. $MRSP = MP_2/MP_1 = \frac{2x_1^{1/2}x_2^{-1/2}}{2x_1^{-1/2}x_2^{1/2}} = x_1/x_2$.
c. Set $MRSP = \$12/\3 and solve jointly with $80 = 4x_1^{1/2}x_2^{1/2}$, to get $x_1^*=40$ and $x_2^*=10$.
d. $TC(80) = 40 \times \$3 + 10 \times \$12 = \$240$.
- (3) [Long-run profit maximization and supply]
a. $AC = TC/q = 0.01q^2 - 0.2q + 5$.
Set $0 = dAC/dq = 0.02q - 0.2$ and solve to get $q_{ES} = 10$.
b. Breakeven price = minimum $AC = AC(q_{ES}) = \$4$.
c. A supply curve shows how much will be produced for any given price. If $P >$ minimum average cost, the profit-maximizing firm will choose an output level where $P=MC(q)$, and if $P <$ minimum average cost, it will produce nothing. So the firm's supply curve is given by the following equations.
If $P \geq \$4$, $P = MC(q) = dTC/dq = 0.03q^2 - 0.4q + 5$.
If $P \leq \$4$, $q=0$ (firm shuts down).
d. Long-run industry supply curve is a horizontal line at minimum AC :



- (4) [Welfare effects of international trade]
- Set $Q_D = Q_S$ and solve to get $P^* = \$4$ and $Q^* = 60$.
 - With international trade, $P_W = \$7$. Substituting into demand and supply gives $Q_D = 30$ and $Q_S = 120$, so the country EXPORTS $120 - 30 = 90$ units.



- Consumer surplus decreases by \$135, the area of the small trapezoid.
- Producer surplus increases by \$270, the area of the large trapezoid.
- The country as a whole gains $\$270 - \$135 = \$135$, the area of the green triangle.

IV. Critical thinking

(Same as Version A above.)

[end of answer key]