

EXAMINATION #1 ANSWER KEY
“Mathematical Tools”

Version A

I. Multiple choice

- (1)c. (2)c. (3)d. (4)e. (5)c. (6)e. (7)a. (8)c. (9)e. (10)a.
(11)e (12)f. (13)b. (14)c. (15)a.

II. Short answer

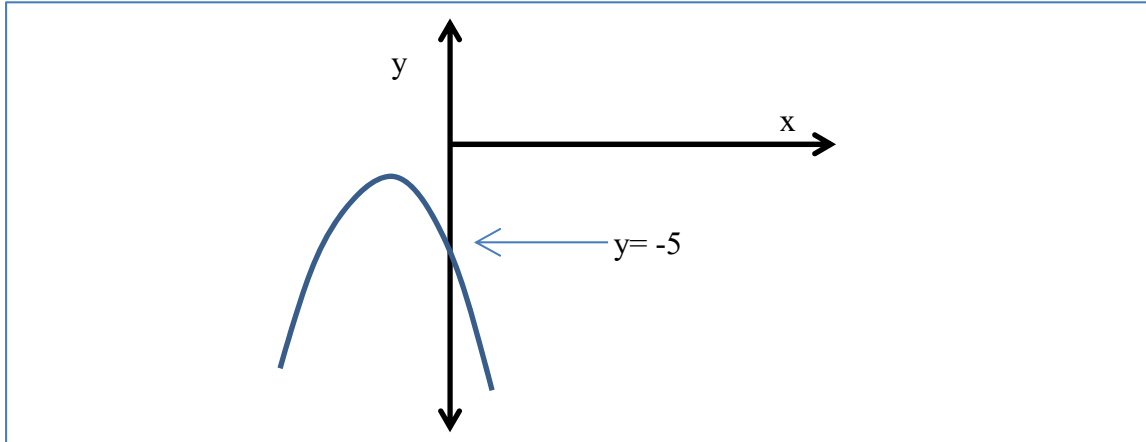
- (1) a. decrease b. 2 percent, using elasticity since change is given in percent.
(2) a. increase b. 3.4 units, using derivatives since changes are given in units.
(3) a. increase b. 3 percent, using approximation rule for products.
(4) a. increase b. 2 percent, using approximation rule for ratios.
(5) a. increase b. 30 units c. decrease d. 15 units.
(6) a. down b. $-5/2 = -2.5 = -\frac{\partial y/\partial x_2}{\partial y/\partial x_1}$.

III. Problems

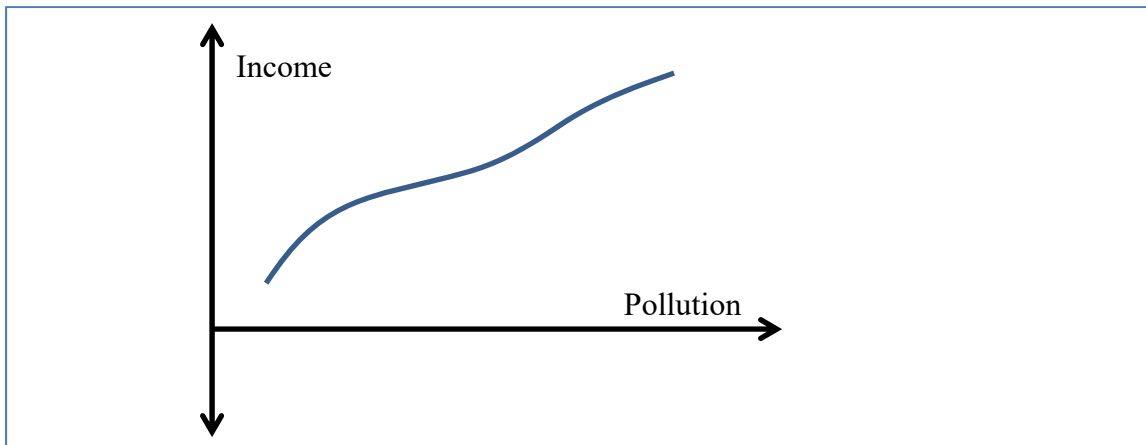
- (1) a. $dy/dx = -4x + 20$. b. Set $dy/dx=0$ and solve to get $x^* = 5$.
c. The function slopes up if $dy/dx = -4x + 20 > 0$, which implies $x < 5$.
The function slopes down if $dy/dx = -4x + 20 < 0$, which implies $x > 5$.
d. $y^* = f(x^*) = f(5) = 53$.
- (2) a. $\varepsilon_1 = \frac{\partial y}{\partial x_1} \frac{x_1}{y} = \frac{2x_1}{x_1-3}$. b. $\varepsilon_2 = \frac{\partial y}{\partial x_2} \frac{x_2}{y} = 4$.
- (3) a. $\frac{\partial y}{\partial x_1} = 3(x_1 - 2)^2 (x_2 + 4)^5$ b. $\frac{\partial y}{\partial x_2} = (x_1 - 2)^3 5(x_2 + 4)^4$
c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{5(x_1-2)}{3(x_2+4)}$.
- (4) a. $\frac{\partial y}{\partial x_1} = 2x_1^{-2}$ b. $\frac{\partial y}{\partial x_2} = 3x_2^{-2}$ c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{3x_2^{-2}}{2x_1^{-2}} = \frac{3}{2} \left(\frac{x_1}{x_2}\right)^2$.

IV. Critical thinking

- (1) We are given the profit function $y = -3x^2 - 2x - 5$. To maximize this function, find the derivative $\frac{dy}{dx} = -6x - 2$. For all non-negative values of x , this derivative is negative and the profit function slopes down. So profit is maximized at the smallest possible value of x , that is, $x^*=0$. At that value of output, profit = $y = -5$. A graph of the profit function is shown below.



- (2) Suppose income is on the vertical axis and pollution is on the horizontal axis. Then the slope of the level curves = $-\frac{\partial U/\partial P}{\partial U/\partial I} = -\frac{\text{negative}}{\text{positive}} = \text{positive}$. So the level curves of $f(I,P)$ must slope UP. (Note that if income is on the horizontal axis and pollution is on the vertical axis, the level curves still slope up.) A graph of a typical level curve is shown below.



Version B

I. Multiple choice

- (1)b. (2)e. (3)b. (4)c. (5)b. (6)e. (7)a. (8)b. (9)b. (10)c.
(11)c (12)d. (13)a. (14)d. (15)d.

II. Short answer

- (1) a. increase b. 6 units, using derivative since change is given in units.
(2) a. increase b. 13 percent, using elasticity since changes are given in percent.
(3) a. decrease b. 2 percent, using approximation rule for products.
(4) a. decrease b. 3 percent, using approximation rule for ratios.
(5) a. increase b. 18 units c. decrease d. 6 units.
(6) a. down b. $-1/3 = -\frac{\partial y/\partial x_2}{\partial y/\partial x_1}$.

III. Problems

- (1) a. $dy/dx = 4x - 12$. b. Set $dy/dx=0$ and solve to get $x^* = 3$.
c. The function slopes up if $dy/dx = 4x - 12 > 0$, which implies $x > 3$.
The function slopes down if $dy/dx = 4x - 12 < 0$, which implies $x < 3$.
d. $y^* = f(x^*) = f(3) = -13$.
- (2) a. $\epsilon_1 = \frac{\partial y}{\partial x_1} \frac{x_1}{y} = 2$. b. $\epsilon_2 = \frac{\partial y}{\partial x_2} \frac{x_2}{y} = \frac{3x_2}{x_2 - 5}$.
- (3) a. $\frac{\partial y}{\partial x_1} = x_1^{-0.5}$ b. $\frac{\partial y}{\partial x_2} = 2x_2^{-0.5}$ c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{2x_2^{-0.5}}{x_1^{-0.5}} = 2\left(\frac{x_1}{x_2}\right)^{0.5}$.
- (4) a. $\frac{\partial y}{\partial x_1} = 3(x_1 + 4)^2 (x_2 - 1)^5$ b. $\frac{\partial y}{\partial x_2} = (x_1 + 4)^3$
c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{(x_1 + 4)}{3(x_2 - 1)}$.

IV. Critical thinking

(Same as version A.)

[end of answer key]