ECON 173 - Intermediate Microeconomic Analysis
Drake University, Fall 2019
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Signature: _____

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EXAMINATION #1 VERSION B
"Mathematical Tools"
September 3, 2019

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [2 pts each—30 pts total]

(1) Suppose the derivative of the function $y = f(x)$ equals -2 at a particular value of x. At that point, the graph of the function is
a. upward-sloping.
b. downward-sloping.
c. vertical.
d. horizontal.
e. cannot be determined from the information given.

(2) Suppose $y = -2/x^3$. Then the derivative of y with respect to x is given by
a. $dy/dx = -6$.
b. $dy/dx = -6x$.
c. $dy/dx = 6x$.
d. $dy/dx = 6/x^2$.
e. $dy/dx = 6/x^4$.

(3) Suppose $y = 2x + x^3$. Then the derivative of y with respect to x is given by
a. $dy/dx = 5$.
b. $dy/dx = 2 + 3x^2$.
c. $dy/dx = 3x^3$.
d. $dy/dx = 5x$.
e. $dy/dx = 2x + 3x$.

(4) Suppose $y = (6-2x)^3$. Then the derivative of y with respect to x is given by
a. $dy/dx = 3$.
b. $dy/dx = 3x$.
c. $dy/dx = -6\,(6-2x)^2$.
d. $dy/dx = 3\,(6-2x)^2$.
e. none of the above.

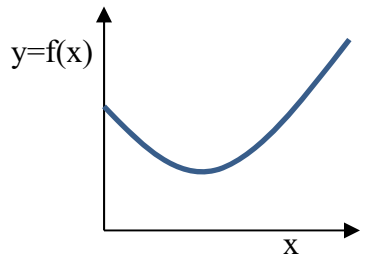
(5) Suppose $y = x^{4/3}$. Then the derivative of y with respect to x is given by
a. $dy/dx = 4x/3$.
b. $dy/dx = 4x^{1/3}/3$.
c. $dy/dx = x^{-1/3}$.
d. $dy/dx = (4/3)\,x^3$.
e. none of the above.

(6) Suppose $y = 5\,(2x+3)^2$. Then the derivative of y with respect to x is
a. $dy/dx = 5$.
b. $dy/dx = 60$.
c. $dy/dx = (2x+3)^2$.
d. $dy/dx = 10\,(2x+3)$.
e. $dy/dx = 20\,(2x+3)$.

(7) Which of the following functions has constant slope (or derivative)?
a. $y = 2 + 4\,x$.
b. $y = 2 + (4/x)$.
c. $y = 2 + 2x + 0.2\,x^4$.
d. $y = 2\,x^{-4}$.
e. $y = \ln(4x)$.
f. $y = \exp(4x)$.

(8) If x increases by 4 percent, then ln(x) increases by about
a. 0.04 percent.
b. 0.04 units.
c. ln(4), or about 1.386 units.
d. 4 percent.
e. 4 units.

The next question refers to the following graph of $y = f(x)$.



(9) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at
a. no point on the graph.
b. one point on the graph.
c. two points on the graph.
d. three points on the graph.
e. four points on the graph.
f. more than four points on the graph.

(10) Suppose we wish to maximize the function $y = f(x)$, which is continuously differentiable. Assuming there are no restrictions on the possible values of x, the maximizing value x^* must satisfy
a. $x^* = 0$.
b. $f(x^*) = 0$.
c. $df/dx = 0$, if $x = x^*$.
d. $d^2f/dx^2 = 0$, if $x = x^*$.
e. All of the above.

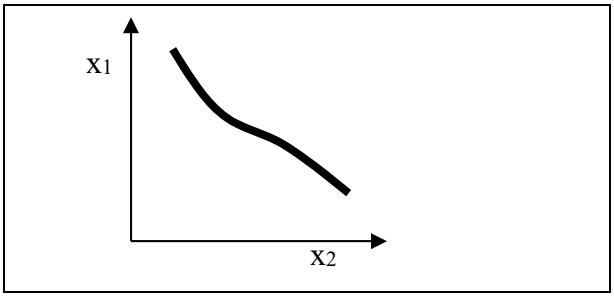
(11) If y is proportional to x, then the elasticity of y with respect to x equals
a. zero.
b. one-half.
c. one.
d. two.
e. x.
f. minus one.
g. cannot be determined from information given.

(12) Consider the following functions. Which has constant elasticity?
a. $y = 2 + 4\,x$.
b. $y = 2 + (4/x)$.
c. $y = 2 + 2x + 0.2\,x^4$.
d. $y = 2\,x^{-4}$.
e. $y = \ln(4x)$.
f. $y = \exp(4x)$.

(13) A straight line has constant
a. slope.
b. elasticity.
c. both of the above.
d. none of the above.

(14) Suppose y depends on both x_1 and x_2, so that $y = f(x_1, x_2)$. By definition, the partial derivative of y with respect to x_1 is the ratio of the change in y to the change in x_1 when x_2
a. equals x_1.
b. changes so as to keep y constant.
c. equals zero.
d. is held constant.

The next question refers to the following graph of a level curve, or contour, of the function $y = f(x_1,x_2)$.



(15) As we move along this curve up and to the left, what remains unchanged?
a. x_1 .
b. x_2 .
c. both x_1, and x_2 .
d. y.
e. the marginal rate of substitution.
f. All of the above remain unchanged.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals 2 at a particular value of x. Moreover, the elasticity of y with respect to x equals 0.6 . Further suppose that x increases by 3 *units*. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

b. By about how much?

units

(2) [4 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 2$, and $\partial y/\partial x_2 = 0.5$, and that the partial elasticities are $\varepsilon_1 = 3$ and $\varepsilon_2 = 0.2$. Further suppose that x_1 increases by 4 *percent* and simultaneously x_2 increases by 5 *percent*. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

b. By about how much?

percent

(3) [4 pts] Total revenue equals price times quantity. Suppose price declines by 5 percent and output quantity increases by 3 percent.

a. Will total revenue *increase* or *decrease*?

b. By about how much? percent

(4) [4 pts] Unit cost equals total cost divided by output quantity. Suppose output quantity increases by 5 percent and total cost increases by 2 percent.

a. Will unit cost *increase* or *decrease*?

b. By about how much? percent

(5) [8 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 3$, and $\partial y/\partial x_2 = 2$. First, suppose that x_2 increases by 9 units but x_1 does not change.

a. Will y *increase* or *decrease*?

b. By about how much? units

Now suppose that x_2 increases by 9 units but we want y to remain constant. To keep y constant, we must change the value of x_1.

c. Must x_1 *increase* or *decrease*?

d. By about how much? units

(6) [4 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 6$, and $\partial y/\partial x_2 = 2$. Now consider a graph of the level curve of this function, with x_1 on the vertical axis and x_2 on the horizontal axis.

a. Does the level curve of the function slope *up* or *down* at that point?

b. Give the slope of the level curve at this point.

III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = 2x^2 - 12x + 5$.

a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

b. Compute the value x* that minimizes this function.

c. For what range of values of x does this function slope up? For what range of values does it slope down?

d. Find the minimum value, y*, of the function itself.

(2) [Partial elasticities: 6 pts] Suppose $y = x_1^2 (x_2-5)^3$.

a. Find an expression for ε_1 , the partial elasticity of y with respect to x_1. The variable y should *not* appear in your answer.

b. Find an expression for ε_2 , the partial elasticity of y with respect to x_2. The variable y should *not* appear in your answer.

(3) [MRS: 12 pts] Suppose $y = f(x_1,x_2) = 2\,x_1^{0.5} + 4\,x_2^{0.5}$. The arguments x_1 and x_2 are strictly positive.

a. Find an expression for the partial derivative of y with respect to x_1.

b. Find an expression for the partial derivative of y with respect to x_2.

c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

(4) [MRS: 12 pts] Suppose $y = f(x_1,x_2) = (x_1 + 4)^3 (x_2 - 1)$. The arguments x_1 and x_2 are strictly positive.

a. Find an expression for the partial derivative of y with respect to x_1 .

b. Find an expression for the partial derivative of y with respect to x_2 .

c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

IV. CRITICAL THINKING: [4 pts] Answer *one* question below (your choice). Circle the question you are answering. Justify your answer and show your work.

(1) Suppose x denotes a firm's output, which cannot be negative, and y denote's the firm's profit, which can be positive or negative. What non-negative value of x maximizes the profit function $y = f(x) = -3x^2 - 2x - 5$? What value of y does it yield? Justify your answer. Sketch the graph of $y = f(x)$. Label the axes.

(2) Suppose $U = f(I,P)$, where U = utility or well-being, P = pollution, and I = income, where P and I are strictly positive. Further suppose $\partial U/\partial I$ is positive, but $\partial U/\partial P$ is negative. Do the level curves of $f(I,P)$ slope *up* or *down*? Justify your answer. Sketch a typical level curve. Label the axes.

[end of exam]