

## EXAMINATION #4 ANSWER KEY

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### Version A

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#### I. Multiple choice

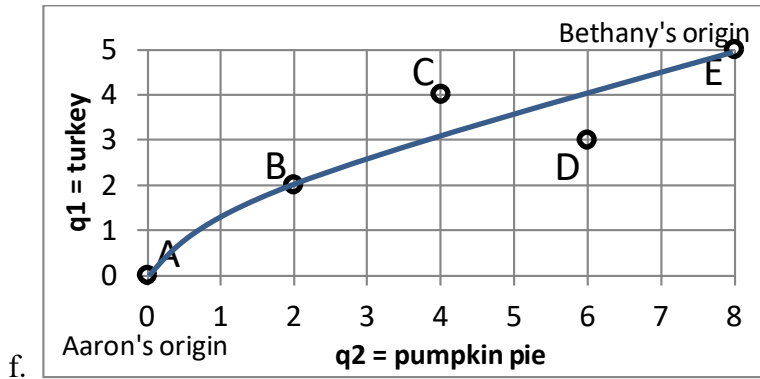
(1)e. (2)b. (3)a. (4)b. (5)b. (6)d. (7)b. (8)b. (9)a. (10)b. (11)a.

#### II. Short answer

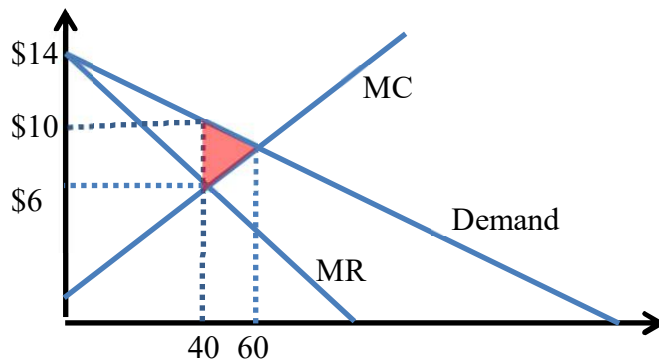
- (1) a. 2 units of food                      b. 1/2 units of health care                      c. slope = -1/2  
d.  $P_{\text{food}} = \$15$ , because slope of each consumer's budget line =  $-P_{\text{food}}/P_{\text{health}} = -1/2$ .
- (2) a. \$12.00                                      b. \$8.00.
- (3) a. 1/2                                              b. 1/10                                              c. 1/20.
- (4) a. \$8                                              b. 12 thousand                                      c. \$0  
d.  $MR = 14 - Q$   
e. MR is straight line with P-intercept = \$14, slope = -1/thousand  
f. \$10                                              g. 8 thousand                                      h. \$8 thousand.
- (5) Note that this game is similar to "Prisoner's Dilemma."  
a. yes, no, yes, yes                      b. no, yes, no, no                      c. no, yes, no, no.

#### III. Problems

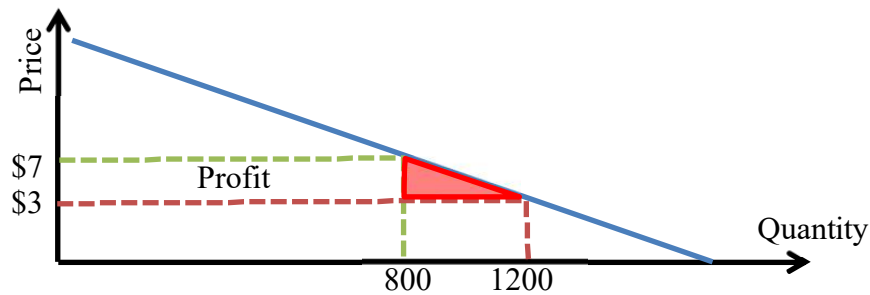
- (1) [Exchange efficiency] Note that Aaron's  $MRS_A = q_1/(2q_2)$  and Bethany's  $MRS_B = q_1/q_2$ .
- a. **Yes**, A is Pareto-efficient, because no one can be made better off without someone else being made worse off. Bethany has everything, so she cannot be made better off. Aaron has nothing, so he cannot be made better off without taking some of Bethany's turkey or pumpkin pie, which would make Bethany worse off. Put simply, since Bethany already has everything, any feasible change would make Bethany worse off.
- b. **Yes**, B is Pareto-efficient, because  $MRS_A = 1/2 = MRS_B$ .
- c. **No**, C is not Pareto-efficient, because  $MRS_A = 1/2 \neq MRS_B = 1/4$ .
- d. **No**, D is not Pareto-efficient, because  $MRS_A = 1/4 \neq MRS_B = 1$ .
- e. **Yes**, E is Pareto-efficient, because no one can be made better off without someone else being made worse off. Aaron has everything, so he cannot be made better off. Bethany has nothing, so she cannot be made better off without taking some of Aaron's turkey or pumpkin pie, which would make Aaron worse off. Put simply, since Aaron already has everything, any feasible change would make Aaron worse off.



- (2) [Monopoly, profit maximization]
- $MC = dTC/dQ = 2 + (Q/10)$ .
  - $AC = TC/Q = 2 + (Q/20)$ .
  - First find total revenue =  $P \times Q = 14Q - (Q^2/10)$ . So  $MR = dTR/dQ = 14 - (2Q/10)$ .
  - Set  $MC = MR$  and solve to get  $Q_M = 40$ .
  - Substitute into demand function:  $P_M = 14 - (40/10) = \$10$ .
  - Profit =  $TR - TC = (40 \times 10) - (2 \times 40 + 40^2/20) = \$240$ .
  - The efficient level of output lies where marginal cost intersects demand (“marginal cost pricing”). Find this quantity by setting  $14 - (Q/10) = 2 + (Q/20)$ , which yields  $Q=60$ . Then find  $MC(60) = 2 + (60/10) = \$8$ . Then evaluate DWL as the area of a triangle: **\$40**.



- (3) [Cournot duopoly]
- $TR_1 = P q_1 = 15q_1 - (q_1^2/100) - (q_1q_2/100)$ .
  - $MR_1 = \partial TR_1(q_1, q_2) / \partial q_1 = 15 - 2q_1/100 - q_2/100$ .
  - Set  $MR_1 = MC = \$3$  and solve to get  $q_1^* = 600 - q_2/2$ .
  - Since  $q_1^* = q_2^*$ ,  $q_1^* = 600 - q_1^*/2$ . Solving yields  $q_1^* = 400 = q_2^*$ .
  - $Q^* = q_1^* + q_2^* = 800$ . Substituting into demand equation:  $P^* = 15 - (800/100) = \$7$ .
  - Profit =  $(P^* \times Q^*) - (AC \times Q^*) = (P^* - AC) \times Q^* = (7 - 3) \times 800 = \$3200$ .
  - The efficient level of output lies where marginal cost intersects demand (“marginal cost pricing”). Find this quantity by setting  $MC = \$3 = P = 15 - (Q/100)$  and solving to get  $Q = 1200$ . Deadweight loss is the area between demand and marginal cost, from the Cournot equilibrium quantity  $Q^*=800$  to the efficient quantity = 1200 (see below). This is the area of a triangle, equal to **\$800**.



#### IV. Critical thinking

(1) Monopoly

$TR = Q \times P = (Q \times 20 Q^{-0.5}) = 20 Q^{0.5}$ . Therefore,  $MR = dTR/dQ = 10 Q^{-0.5}$ .  
 To maximize profit, choose quantity where  $MC=MR$ :  $2 = 10 Q^{-0.5}$   
 or  $2/10 = Q^{-0.5}$  or  $5 = Q^{0.5}$  or  $25 = Q_M$ .  $P_M = 20 (25)^{-0.5} = \$4$ .  
 Profit =  $TR - TC = (25 \times 4) - (25 \times 2) = \$50$ .

(2) Triopoly

Note that MC and demand are the same as in problem (3) above, but now we have three firms instead of two.

$$TR_1 = q_1 \times P = 15q_1 - (q_1^2 + q_1q_2 + q_1q_3)/100.$$

$$\text{Therefore, } MR_1 = \partial TR_1 / \partial q_1 = 15 - (2q_1 + q_2 + q_3)/100.$$

Next, find firm #1's reaction function  $q_1^* = f(q_2, q_3)$ . Set  $MC = MR_1$ :

$$3 = 15 - (2q_1 + q_2 + q_3)/100 \text{ and solve to get } q_1^* = 600 - (q_2 + q_3)/2.$$

Symmetry implies that  $q_1 = q_2 = q_3$ , so substitute:  $q_1 = 600 - (q_1 + q_1)/2$

and solve to get  $q_1 = q_2 = q_3 = 300$ . So  $Q^* = 900$ , and  $P^* = 15 - 900/100 = \$6$ .

### Version B

#### I. Multiple choice

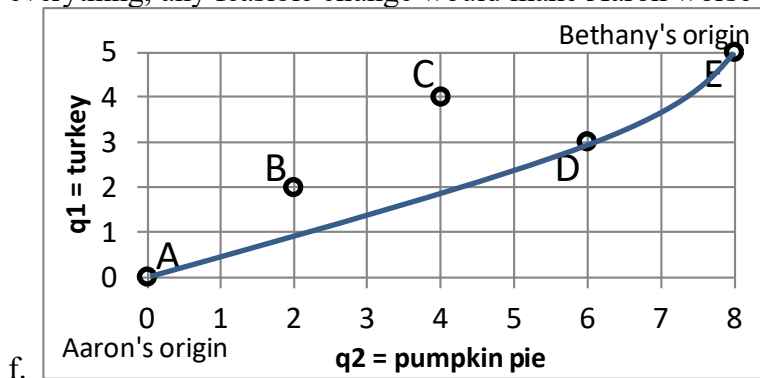
(1)a. (2)e. (3)c. (4)a. (5)c. (6)c. (7)c. (8)c. (9)b. (10)c. (11)d.

#### II. Short answer

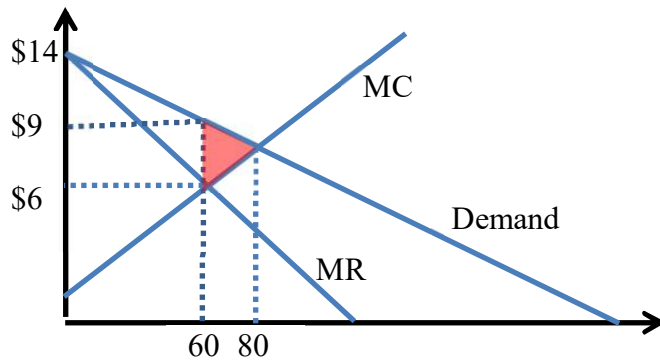
- (1) a. 3 units of food                      b. 1/3 units of health care              c. slope = -1/3  
 d.  $P_{\text{food}} = \$10$ , because slope of each consumer's budget line =  $-P_{\text{food}}/P_{\text{health}} = -1/3$ .
- (2) a. \$12.00                                      b. \$5.00.
- (3) a. 2/3                                              b. 1/3                                              c. 2/15.
- (4) a. \$6                                              b. 10 thousand                              c. \$0  
 d.  $MR = 16 - 2Q$   
 e. MR is straight line with P-intercept = \$16, slope = -2/thousand  
 f. \$10                                              g. 6 thousand                              h. \$12 thousand.
- (5) Note that this game is similar to "Battle of the Sexes."  
 a. yes, no, no, yes                              b. no, no, no, no                              c. yes, no, no, yes.

### III. Problems

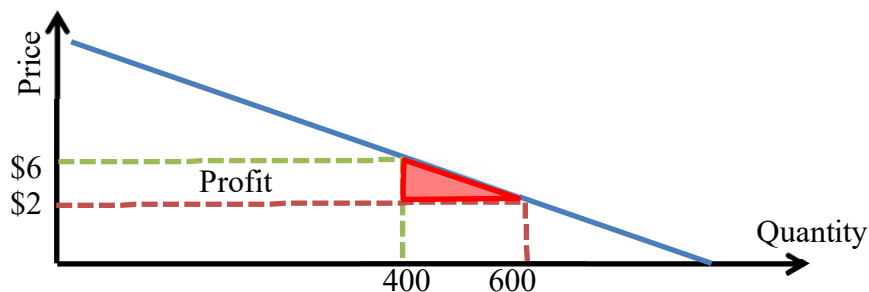
- (1) [Exchange efficiency] Note that Aaron's  $MRS_A = q_1/q_2$  and Bethany's  $MRS_B = q_1/(2q_2)$ .
- Yes**, A is Pareto-efficient, because no one can be made better off without someone else being made worse off. Bethany has everything, so she cannot be made better off. Aaron has nothing, so he cannot be made better off without taking some of Bethany's turkey or pumpkin pie, which would make Bethany worse off. Put simply, since Bethany already has everything, any feasible change would make Bethany worse off.
  - No**, B is not Pareto-efficient, because  $MRS_A = 1 \neq MRS_B = 1/4$ .
  - No**, C is not Pareto-efficient, because  $MRS_A = 1 \neq MRS_B = 1/8$ .
  - Yes**, D is Pareto-efficient, because  $MRS_A = 1/2 = MRS_B$ .
  - Yes**, E is Pareto-efficient, because no one can be made better off without someone else being made worse off. Aaron has everything, so he cannot be made better off. Bethany has nothing, so she cannot be made better off without taking some of Aaron's turkey or pumpkin pie, which would make Aaron worse off. Put simply, since Aaron already has everything, any feasible change would make Aaron worse off.



- (2) [Monopoly, profit maximization]
- $MC = dTC/dQ = (Q/10)$ .
  - $AC = TC/Q = (Q/20)$ .
  - First find total revenue =  $P \times Q = 12Q - (Q^2/20)$ . So  $MR = dTR/dQ = 12 - (Q/10)$ .
  - Set  $MC = MR$  and solve to get  $Q_M = 60$ .
  - Substitute into demand function:  $P_M = 12 - (60/20) = \$9$ .
  - Profit =  $TR - TC = (60 \times 9) - (60^2/20) = \$360$ .
  - The efficient level of output lies where marginal cost intersects demand ("marginal cost pricing"). Find this quantity by setting  $12 - (Q/20) = (Q/10)$ , which yields  $Q=80$ . Then find  $MC(60) = (60/10) = \$6$ . Then evaluate DWL as the area of a triangle: **\$30**.



- (3) [Cournot duopoly]
- $TR_1 = P q_1 = 14q_1 - (q_1^2/50) - (q_1q_2/50)$ .
  - $MR_1 = \partial TR_1(q_1, q_2) / \partial q_1 = 14 - 2q_1/50 - q_2/50$ .
  - Set  $MR_1 = MC = \$2$  and solve to get  $q_1^* = 300 - q_2/2$ .
  - Since  $q_1^* = q_2^*$ ,  $q_1^* = 300 - q_1^*/2$ . Solving yields  $q_1^* = 200 = q_2^*$ .
  - $Q^* = q_1^* + q_2^* = 400$ . Substituting into demand equation:  $P^* = 14 - (400/50) = \$6$ .
  - Profit =  $(P^* \times Q^*) - (AC \times Q^*) = (P^* - AC) \times Q^* = (6 - 2) \times 400 = \$1600$ .
  - The efficient level of output lies where marginal cost intersects demand (“marginal cost pricing”). Find this quantity by setting  $MC = \$2 = P = 14 - (Q/50)$  and solving to get  $Q = 600$ . Deadweight loss is the area between demand and marginal cost, from the Cournot equilibrium quantity  $Q^* = 400$  to the efficient quantity = 600 (see below). This is the area of a triangle, equal to **\$400**.



#### IV. Critical thinking

- Monopoly  
 Same as Version A.
- Triopoly  
 Note that MC and demand are the same as in problem (3) above, but now we have three firms instead of two.  
 $TR_1 = q_1 \times P = 14q_1 - (q_1^2 + q_1q_2 + q_1q_3)/50$ .  
 Therefore,  $MR_1 = \partial TR_1 / \partial q_1 = 14 - (2q_1 + q_2 + q_3)/50$ .  
 Next, find firm #1’s reaction function  $q_1^* = f(q_2, q_3)$ . Set  $MC = MR_1$ :  
 $2 = 14 - (2q_1 + q_2 + q_3)/50$  and solve to get  $q_1^* = 300 - (q_2 + q_3)/2$ .

Symmetry implies that  $q_1 = q_2 = q_3$ , so substitute:  $q_1 = 300 - (q_1 + q_1)/2$   
and solve to get  $q_1 = q_2 = q_3 = 150$ . So  $Q^* = 450$ , and  $P^* = 14 - 450/50 = \$5$ .

[end of answer key]