

EXAMINATION #3 ANSWER KEY

Version A

I. Multiple choice

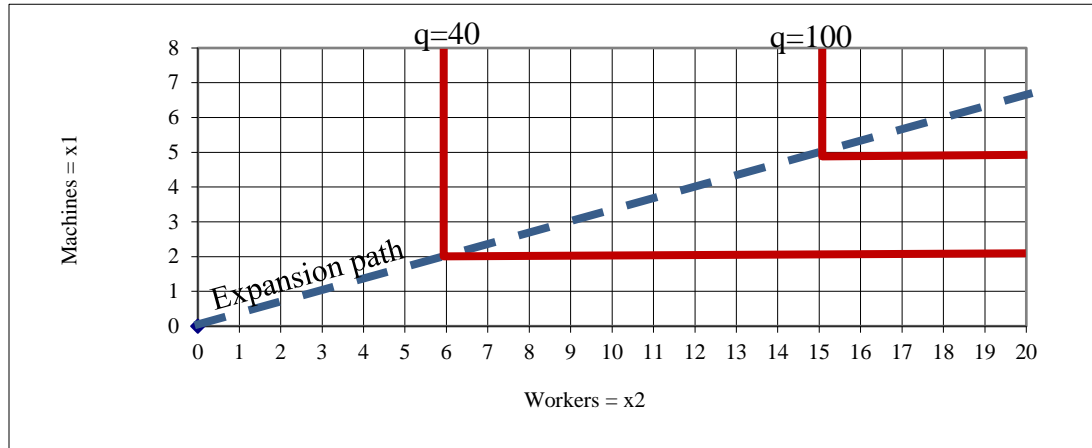
- (1)c. (2)c. (3)c. (4)d. (5)d. (6)c. (7)a. (8)b. (9)b. (10)c.
(11)b. (12)b. (13)c. (14)f.

II. Short answer

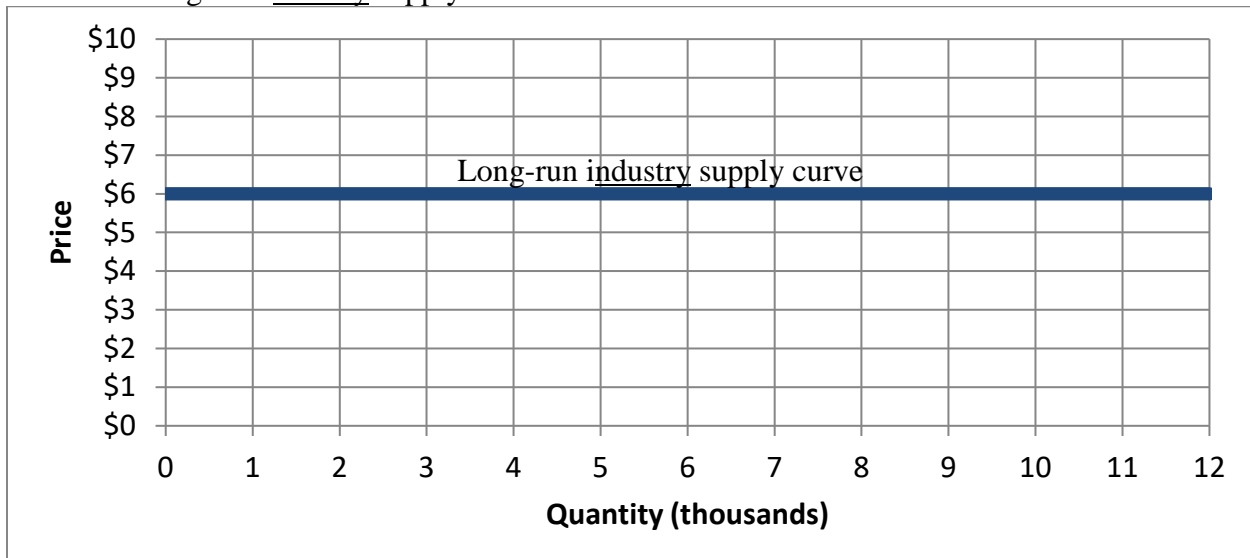
- (1) a. 0 thousand (because price is below shutdown price).
b. 8 thousand (using rule $P=MC$ to find q).
c. 11 thousand (using rule $P=MC$ to find q).
d. \$6 (because breakeven price = $\min(\text{SATC})$).
e. \$2 (because shutdown price = $\min(\text{SAVC})$).
- (2) a. 70 units (set $MC_A(q_A) = MC_B(100-q_A)$ and solve for q_A).
b. 30 units.
c. \$12 (= $MC_A(q_A^*) = MC_B(q_B^*)$).
- (3) a. import. b. 5 million gallons. c. increase.
d. \$8.5 million. e. decrease. f. \$6 million.
g. increase. h. \$2.5 million.

III. Problems

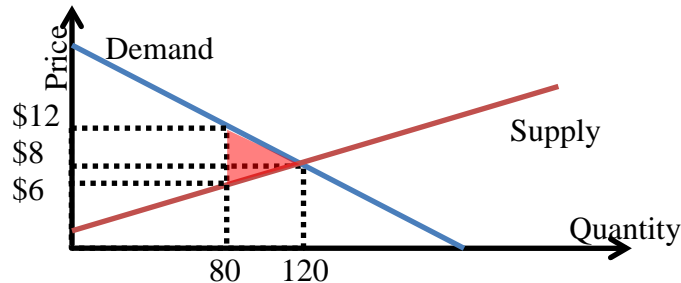
- (1) [Production functions]
- a. $MP_1 = \frac{1}{2}(x_1 + x_2)^{-1/2} = \frac{1}{2(x_1+x_2)^{1/2}}$. YES, there are diminishing returns to input 1, because as x_1 increases (and x_2 is held constant), MP_1 decreases.
- b. $MRSP = MP_2/MP_1 = \frac{\frac{1}{2}(x_1+x_2)^{-1/2}}{\frac{1}{2}(x_1+x_2)^{-1/2}} = 1$. No, this function does NOT have diminishing MRSP, because as x_1 decreases and x_2 increases, MRSP remains constant.
- c. Check returns to scale:
 $f(ax_1, ax_2) = (ax_1 + ax_2)^{1/2} = (a(x_1 + x_2))^{1/2} = a^{1/2}(x_1 + x_2)^{1/2}$,
 $= a^{1/2} q < a q$, for all $a > 1$.
So this production function has DECREASING returns to scale.
- (2) [Fixed-proportions technology]
- a. $x_1 = (x_2/3)$. b. $q = 20 x_1$. c. $q = (20/3) x_2$.
d. $q = \min\{20 x_1, (20/3) x_2\}$.
e.



- (3) [Cost minimization]
- $30 = 5 x_1^{1/2} x_2^{1/2}$ or $6 = x_1^{1/2} x_2^{1/2}$ or $36 = x_1 x_2$.
 - $MRSP = MP_2 / MP_1 = \frac{(5/2) x_1^{1/2} x_2^{-1/2}}{(5/2) x_1^{-1/2} x_2^{1/2}} = x_1 / x_2$.
 - Set $MRSP = \$20 / \5 and solve jointly with $30 = 5 x_1^{1/2} x_2^{1/2}$, to get $x_1^* = 12$ and $x_2^* = 3$.
 - $TC(30) = 12 \times \$5 + 3 \times \$20 = \$120$.
- (4) [Long-run profit maximization and supply]
- $AC = TC/q = (1/12) q^2 - 2q + 18$.
 Set $0 = dAC/dq = (1/6)q - 2$ and solve to get $q_{ES} = 12$.
 - Breakeven price = minimum $AC = AC(q_{ES}) = \$6$.
 - Firm's supply curve is as follows.
 If $P \geq \$6$, $P = MC(q) = dTC/dq = (1/4)q^2 - 4q + 18$.
 If $P < \$6$, $q=0$ (firm shuts down).
 - Long-run industry supply curve is a horizontal line at minimum AC :



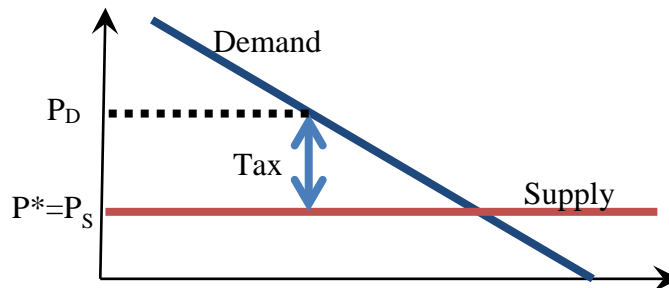
- (5) [Welfare effects of tax or subsidy]
- Set $P_D = P_S$ and solve to get $P^* = \$8$ and $Q^* = 120$.
 - With an excise tax of \$6, $P_D = P_S + 6$. Substituting and solving gives $Q = 80$. It is useful to also compute the new total price paid by buyers, including the tax ($P_D = \$12$), and the new net price received by sellers, excluding the tax ($P_S = \$6$).



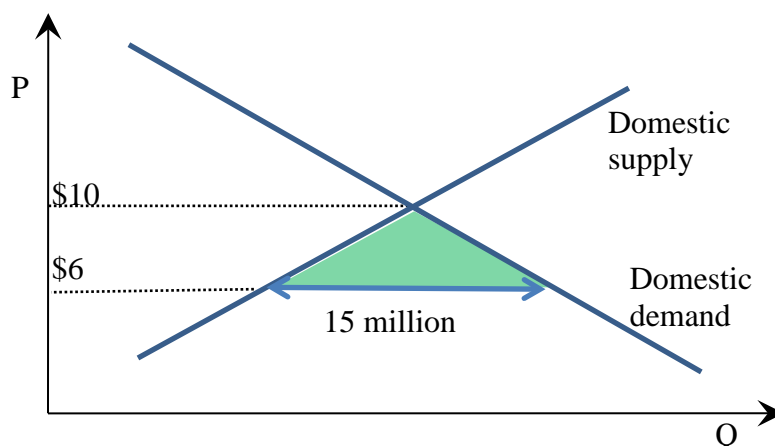
- Consumer surplus decreases by \$400, the area of the trapezoid between \$12 and \$8.
- Producer surplus decreases by \$200, the area of the trapezoid between \$6 and \$8.
- Although the government collects \$480 in tax revenue, this is less than the combined decreases of consumer and producer surplus. The loss to society as a whole (also called “deadweight loss” or “excess burden of the tax”) is \$120.

IV. Critical thinking

(1) If supply is perfectly elastic, a tax raises the total price (including the tax) paid by buyers, but has no effect on the net price (excluding the tax) received by sellers. Also, consumer surplus is reduced by the tax, but producer surplus is still zero and unchanged as a result of the tax. So buyers bear the entire burden of the tax. See graph below.



(2) We are given that the price of tee-shirts fell from \$10 to \$6, and that 15 million tee-shirts are imported. Clearly consumers win from this change and producers lose, but overall welfare increases (as usual for international trade). With the information given, it is not possible to compute separately the gain in consumer surplus or the loss of producer surplus. Nevertheless, we can compute the area of the triangle that measures the net gain to society. That area is $(1/2) \times (\$10 - \$6) \times 15 \text{ million} = \30 million . See graph below.



Version B

I. Multiple choice

- (1)b. (2)b. (3)d. (4)c. (5)b. (6)a. (7)b. (8)a. (9)d. (10)d.
 (11)c. (12)c. (13)d. (14)e.

II. Short answer

- (1) a. 9 thousand (using rule $P=MC$ to find q).
 b. 0 thousand (because price is below shutdown price).
 c. 7 thousand (using rule $P=MC$ to find q).
 d. \$7 (because breakeven price = $\min(\text{SATC})$).
 e. \$4 (because shutdown price = $\min(\text{SAVC})$).
 (2) a. 20 units (set $MC_A(q_A) = MC_B(100-q_A)$ and solve for q_A).
 b. 80 units.
 c. \$7 (= $MC_A(q_A^*) = MC_B(q_B^*)$).
 (3) a. export. b. 5 million gallons. c. decrease.
 d. \$7.5 million. e. increase. f. \$10 million.
 g. increase. h. \$2.5 million.

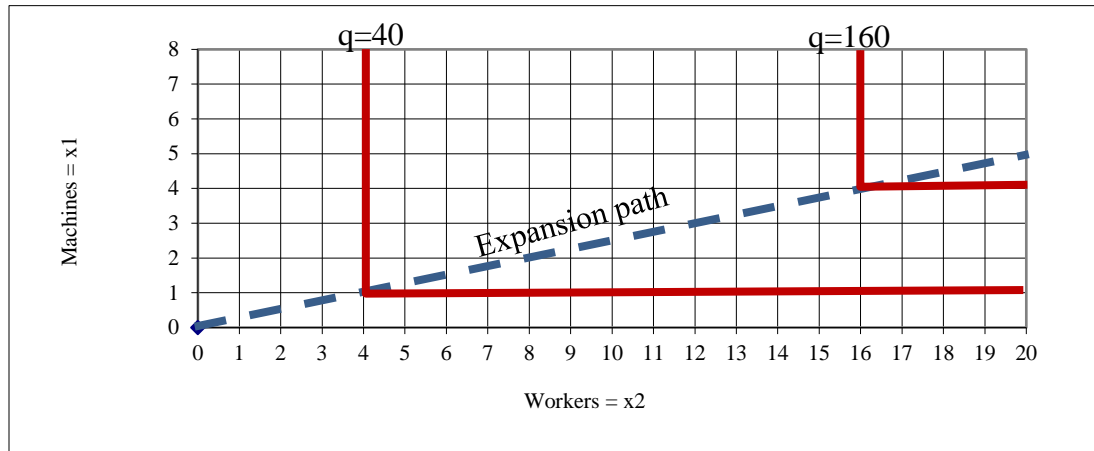
III. Problems

- (1) [Production functions]
 a. $MP_1 = 2(x_1 + x_2)$. No, there are no diminishing returns to input 1, because as x_1 increases (and x_2 is held constant), MP_1 increases.
 b. $MRSP = MP_2/MP_1 = \frac{2(x_1+x_2)}{2(x_1+x_2)} = 1$. No, this function does NOT have diminishing $MRSP$, because as x_1 decreases and x_2 increases, $MRSP$ remains constant.
 c. Check returns to scale:

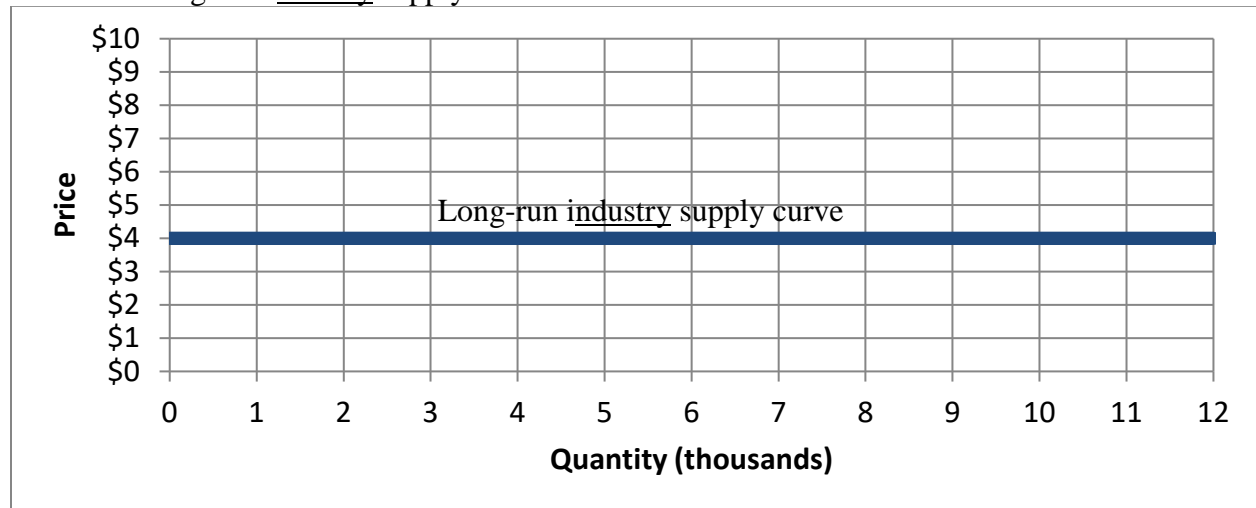
$$f(ax_1, ax_2) = (ax_1 + ax_2)^2 = (a(x_1 + x_2))^2 = a^2(x_1 + x_2)^2,$$

$$= a^2 q > a q, \text{ for all } a > 1.$$
 So this production function has INCREASING returns to scale.
 (2) [Fixed-proportions technology]

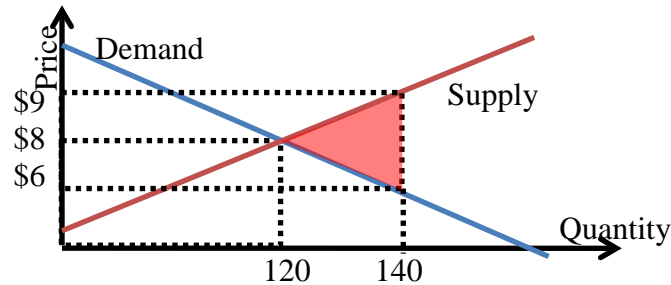
- a. $x_1 = (x_2/4)$. b. $q = 40 x_1$. c. $q = (40/4) x_2 = 10 x_2$.
 d. $q = \min\{40 x_1, 10 x_2\}$.
 e.



- (3) [Cost minimization]
 a. $30 = 5 x_1^{1/2} x_2^{1/2}$ or $6 = x_1^{1/2} x_2^{1/2}$ or $36 = x_1 x_2$.
 b. $MRSP = MP_2/MP_1 = \frac{(5/2) x_1^{1/2} x_2^{-1/2}}{(5/2) x_1^{-1/2} x_2^{1/2}} = x_1/x_2$.
 c. Set $MRSP = \$9/\1 and solve jointly with $30 = 5 x_1^{1/2} x_2^{1/2}$, to get $x_1^*=18$ and $x_2^*=2$.
 d. $TC(30) = 18 \times \$1 + 2 \times \$9 = \$36$.
 (4) [Long-run profit maximization and supply]
 a. $AC = TC/q = 0.01 q^2 - 0.4 q + 8$.
 Set $0 = dAC/dq = 0.02 q - 0.4$ and solve to get $q_{ES} = 20$.
 b. Breakeven price = minimum AC = $AC(q_{ES}) = \$4$.
 c. Firm's supply curve is as follows.
 If $P \geq \$4$, $P = MC(q) = dTC/dq = 0.03 q^2 - 0.8 q + 8$.
 If $P \leq \$4$, $q=0$ (firm shuts down).
 d. Long-run industry supply curve is a horizontal line at minimum AC:



- (5) [Welfare effects of tax or subsidy]
- Set $P_D = P_S$ and solve to get $P^* = \$8$ and $Q^* = 120$.
 - With an subsidy of \$3, $P_D + 3 = P_S$. Substituting and solving gives $Q = 140$. It is useful to also compute the new total price paid by buyers, excluding the subsidy ($P_D = \$6$), and the new net price received by sellers, including the subsidy ($P_S = \$9$).



- Consumer surplus increases by \$260, the area of the trapezoid between \$8 and \$6.
- Producer surplus increases by \$130, the area of the trapezoid between \$8 and \$9.
- The government pays \$420 in direct costs, which is less than the combined increases of consumer and producer surplus. The loss to society as a whole (also called “deadweight loss”) is \$30.

IV. Critical thinking

(Same as Version A above.)

[end of answer key]