

## EXAMINATION #1 ANSWER KEY “Mathematical Tools”

### Version A

#### I. Multiple choice

- (1)b. (2)b. (3)c. (4)d. (5)d. (6)e. (7)c. (8)b. (9)c. (10)c.  
 (11)c. (12)f. (13)a. (14)b. (15)b.

#### II. Short answer

- (1) a. increase                      b. 6 units, using derivative since change is given in units.  
 (2) a. increase                      b. 7 percent, using elasticities since changes are given in percent.  
 (3) a. increase                      b. 4 %, using approximation rule for products.  
 (4) a. increase                      b. 3 %, using approximation rule for ratios.  
 (5) a. decrease                      b. 12 units                      c. increase                      d. 4 units.  
 (6) a. down                              b.  $-1/5 = -0.2 = \frac{\partial y / \partial x_2}{\partial y / \partial x_1}$ .

#### III. Problems

- (1) a.  $dy/dx = 6x-12$ .              b.  $x^* = 2$ .  
 c. The function slopes up if  $dy/dx = 6x-12 > 0$ , which implies  $x > 2$ .  
 The function slopes down if  $dy/dx = 6x-12 < 0$ , which implies  $x < 2$ .  
 d.  $y^* = f(x^*) = f(2) = 8$ .  
 (2) a.  $\varepsilon_1 = 2$ .                      b.  $\varepsilon_2 = \frac{2x_2}{(x_2-5)}$ .  
 (3) a.  $\frac{\partial y}{\partial x_1} = 2x_1^{-0.5}$               b.  $\frac{\partial y}{\partial x_2} = 3x_2^{-0.5}$               c.  $MRS = \frac{\partial y / \partial x_2}{\partial y / \partial x_1} = \frac{3x_2^{-0.5}}{2x_1^{-0.5}} = \frac{3}{2} \left( \frac{x_1}{x_2} \right)^{0.5}$ .  
 (4) a.  $\frac{\partial y}{\partial x_1} = 2(x_1 - 5)(x_2 + 3)^3$               b.  $\frac{\partial y}{\partial x_2} = (x_1 - 5)^2 3(x_2 + 2)^2$   
 c.  $MRS = \frac{\partial y / \partial x_2}{\partial y / \partial x_1} = \frac{3(x_1-5)}{2(x_2+2)}$ .

#### IV. Critical thinking

- (1) We are given that  $f(x) = b(x) - c(x)$ . The first-order necessary condition implies that  
 $0 = \frac{df}{dx} = \frac{db}{dx} - \frac{dc}{dx}$ , using the rule for the derivative of a sum. So when  $f(x)$  is  
 maximized,  $\frac{db}{dx} = \frac{dc}{dx}$ . [In economic terminology, “when net benefit is maximized,  
 marginal benefit equals marginal cost.”]  
 (2) We are given that  $P \times Q = a$ , so therefore  $Q = aP^{-1}$ . The elasticity of  $Q$  with respect  
 to  $P$  is  $\varepsilon = \frac{dQ}{dP} \frac{P}{Q} = -aP^{-2} \frac{P}{aP^{-1}} = -1$ . Clearly, the elasticity does not depend on  $a$ .  
 [In economic terminology, “if revenue is constant, regardless of price, then demand is  
 unitary-elastic.”]

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## Version B

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### I. Multiple choice

- (1)a. (2)e. (3)b. (4)c. (5)b. (6)d. (7)a. (8)e. (9)d. (10)b.  
(11)d. (12)d. (13)b. (14)d. (15)a.

### II. Short answer

- (1) a. increase b. 3 %, using elasticity since change is given in percent.  
(2) a. increase b. 7 units, using derivative since changes are given in units.  
(3) a. increase b. 5 %, using approximation rule for products.  
(4) a. increase b. 1 %, using approximation rule for ratios.  
(5) a. increase b. 12 units c. decrease d. 6 units.  
(6) a. down b. -3.

### III. Problems

- (1) a.  $dy/dx = -4x + 20$ . b.  $x^* = 5$ .  
c. The function slopes up if  $dy/dx = -4x+20 > 0$ , which implies  $x < 5$ .  
The function slopes up if  $dy/dx = -4x+20 < 0$ , which implies  $x > 5$ .  
d.  $y^* = f(x^*) = f(5) = 53$ .
- (2) a.  $\varepsilon_1 = \frac{3x_1}{x_1-2}$ . b.  $\varepsilon_1 = 2$ .
- (3) a.  $\frac{\partial y}{\partial x_1} = 3(x_1 + 2)^2(x_2 - 3)^3$  b.  $\frac{\partial y}{\partial x_2} = (x_1 + 2)^3 4(x_2 - 3)^3$   
c.  $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{4(x_1+2)}{3(x_2-3)}$ .
- (4) a.  $\frac{\partial y}{\partial x_1} = 4 x_1^{-1.5}$  b.  $\frac{\partial y}{\partial x_2} = x_2^{-1.5}$  c.  $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{x_2^{-1.5}}{4 x_1^{-1.5}} = \frac{1}{4} \left(\frac{x_1}{x_2}\right)^{1.5}$ .

### IV. Critical thinking

(Same as version A.)

[end of answer key]