

EXAMINATION #1 VERSION B
“Mathematical Tools”
September 4, 2018

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [2 pts each—30 pts total]

(1) Suppose the derivative of the function $y = f(x)$ equals 0.5 at a particular value of x . At that point, the graph of the function is

- a. upward-sloping.
- b. downward-sloping.
- c. vertical.
- d. horizontal.
- e. cannot be determined from the information given.

(2) Suppose $y = -5/x$. Then the derivative of y with respect to x is given by

- a. $dy/dx = -5$.
- b. $dy/dx = -5x$.
- c. $dy/dx = 5x$.
- d. $dy/dx = 5/x$.
- e. $dy/dx = 5/x^2$.

(3) Suppose $y = 5x - x^2$. Then the derivative of y with respect to x is given by

- a. $dy/dx = 5$.
- b. $dy/dx = 5 - 2x$.
- c. $dy/dx = 3x$.
- d. $dy/dx = 5x - x^2$.
- e. $dy/dx = (5/2)x^2 - (1/3)x^3$.

(4) Suppose $y = (2+x)^2$. Then the derivative of y with respect to x is given by

- a. $dy/dx = 2$.
- b. $dy/dx = 2x$.
- c. $dy/dx = 2(2+x)$.
- d. $dy/dx = (2 + 2x)$.
- e. none of the above.

(5) Suppose $y = x^{1/3}$. Then the derivative of y with respect to x is given by

- a. $dy/dx = x/3$.
- b. $dy/dx = x^{-2/3}/3$.
- c. $dy/dx = x^{2/3}$.
- d. $dy/dx = (-2/3)x^3$.
- e. none of the above.

(6) Suppose $y = 2(4x+5)^3$. Then the derivative of y with respect to x is

- a. $dy/dx = 2$.
- b. $dy/dx = (4x+5)^2$.
- c. $dy/dx = 6(4x+5)^2$.
- d. $dy/dx = 24(4x+5)^2$.
- e. $dy/dx = 2(4x+5)^2$.

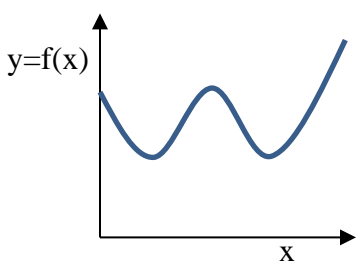
(7) Which of the following functions has constant slope (or derivative)?

- a. $y = 5 + 3x$.
- b. $y = 5 + (3/x)$.
- c. $y = 5 + 2x + 0.5x^3$.
- d. $y = 5x^{-3}$.
- e. $y = \ln(3x)$.
- f. $y = \exp(3x)$.

(8) If x increases by 5 percent, then $\ln(x)$ increases by about

- a. $\ln(5)$, or about 1.609 units.
- b. 5 percent.
- c. 5 units.
- d. 0.05 percent.
- e. 0.05 units.

The next question refers to the following graph of $y = f(x)$.



(9) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at

- a. no point on the graph.
- b. one point on the graph.
- c. two points on the graph.
- d. three points on the graph.
- e. four points on the graph.
- f. more than four points on the graph.

(10) Suppose we wish to minimize the function $y = f(x)$, which is continuously differentiable. Assuming there are no restrictions on the possible values of x , the minimizing value x^* must satisfy

- a. $f(x^*) = 0$.
- b. $df/dx = 0$, if $x = x^*$.
- c. $d^2f/dx^2 = 0$, if $x = x^*$.
- d. $x^* = 0$.
- e. All of the above.

(11) If y is inversely proportional to x (that is, if $y = c/x$, where c is an unknown constant) then the elasticity of y with respect to x equals

- a. zero.
- b. one-half.
- c. one.
- d. minus one.
- e. c .
- f. cannot be determined from information given.

(12) Consider the following functions. Which has constant elasticity?

- a. $y = 5 + 3x$.
- b. $y = 5 + (3/x)$.
- c. $y = 5 + 2x + 0.5x^3$.
- d. $y = 5x^{-3}$.
- e. $y = \ln(3x)$.
- f. $y = \exp(3x)$.

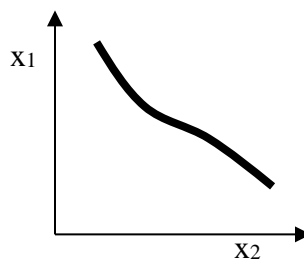
(13) A straight line does *not* have constant

- a. slope.
- b. elasticity.
- c. It has neither constant slope nor constant elasticity.
- d. It has both constant slope and constant elasticity.

(14) Suppose y depends on both x_1 and x_2 , so that $y = f(x_1, x_2)$. By definition, the partial derivative of y with respect to x_1 is the ratio of the change in y to the change in x_1 when x_2

- a. equals x_1 .
- b. changes so as to keep y constant.
- c. equals zero.
- d. is held constant.

The next question refers to the following graph of a level curve, or contour, of the function $y = f(x_1, x_2)$.



(15) According to this graph, if x_2 decreases and y is to be held constant, then x_1 must

- a. increase.
- b. decrease.
- c. remain constant.
- d. equal zero.
- e. cannot be determined from the information given.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals 2 at a particular value of x . Moreover, the elasticity of y with respect to x equals 0.6. Further suppose that x increases by 5 percent. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y increase or decrease?

percent

b. By about how much?

(2) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 2$, and $\partial y / \partial x_2 = 0.5$, and that the partial elasticities are $\epsilon_1 = 3$ and $\epsilon_2 = 0.25$. Further suppose that x_1 increases by 3 units and simultaneously x_2 increases by 2 units. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y increase or decrease?

units

b. By about how much?

(3) [4 pts] GDP equals the number of workers times productivity. Suppose the number of workers increases by 2 percent and productivity increases by 3 percent.

a. Will GDP *increase* or *decrease*?

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b. By about how much?

percent

(4) [4 pts] Per-capita income is defined as total income divided by population. Suppose total income increases by 3 percent and population increases by 2 percent.

a. Will per-capita income *increase* or *decrease*?

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b. By about how much?

percent

(5) [8 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 2$, and $\partial y / \partial x_2 = 3$. First, suppose that x_2 increases by 4 units but x_1 does not change.

a. Will y *increase* or *decrease*?

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b. By about how much?

units

Now suppose that x_2 increases by 4 units but we want y to remain constant. To keep y constant, we must change the value of x_1 .

c. Must x_1 *increase* or *decrease*?

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d. By about how much?

units

(6) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 5$, and $\partial y / \partial x_2 = 15$. Now consider a graph of the level curve of this function, with x_1 on the vertical axis and x_2 on the horizontal axis.

a. Does the level curve of the function slope *up* or *down* at that point?

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b. Give the slope of the level curve at this point.

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III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = -2x^2 + 20x + 3$.

a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

b. Compute the value x^* that maximizes this function.

c. For what range of values of x does this function slope up? For what range of values does it slope down?

d. Find the maximum value, y^* , of the function itself.

(2) [Partial elasticities: 6 pts] Suppose $y = (x_1 - 2)^3 x_2^2$.

- a. Find an expression for ε_1 , the partial elasticity of y with respect to x_1 . The variable y should *not* appear in your answer.

- b. Find an expression for ε_2 , the partial elasticity of y with respect to x_2 . The variable y should *not* appear in your answer.

(3) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = (x_1 + 2)^3 (x_2 - 3)^4$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the partial derivative of y with respect to x_1 .

- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the $|\text{slope}|$ of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

(4) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = -8x_1^{-0.5} - 2x_2^{-0.5}$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the partial derivative of y with respect to x_1 .

- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

IV. CRITICAL THINKING: [4 pts] Answer *one* question below (your choice). Circle the question you are answering. Justify your answer and show your work.

- (1) Let the function $f(x)$ represent net benefit to society as a function of some variable x , which might represent the amount of output of some good or service. Now $f(x)$ is itself the difference of two other functions $b(x)$ and $c(x)$, representing benefits and costs of x , respectively, so that $f(x) = b(x) - c(x)$. When $f(x)$ is maximized, what must be the relationship between the derivatives db/dx and dc/dx ?
- (2) Suppose $P \times Q = a$, where a is some constant, for all values of P and Q . Find the elasticity of Q with respect to P . Does the elasticity depend on the value of a ?

[end of exam]