ECON 173 - Intermediate Microeconomic Analysis
Drake University, Fall 2018
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EXAMINATION #1 VERSION A
"Mathematical Tools"
September 4, 2018

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [2 pts each—30 pts total]

(1) Suppose the derivative of the function y = f(x) equals -2 at a particular value of x . At that point, the graph of the function is
a. upward-sloping.
b. downward-sloping.
c. vertical.
d. horizontal.
e. cannot be determined from the information given.

(2) Suppose y = 5 + (2/x). Then the derivative of y with respect to x is given by the formula
a. dy/dx = 2.
b. dy/dx = $-2/x^2$.
c. dy/dx = $2/x^2$.
d. dy/dx = 5x .
e. none of the above.

(3) Suppose $y = 3x^2 + 5x + 7$. Then the derivative of y with respect to x is
a. dy/dx = 6 .
b. dy/dx = 5 .
c. dy/dx = 6x + 5 .
d. dy/dx = 3x + 5 .
e. dy/dx = 5x + 7 .
f. dy/dx = $6x^2 + 6x + 7$.

(4) Suppose $y = (3x-5)^2$. Then the derivative of y with respect to x is given by
a. dy/dx = 2.
b. dy/dx = 3.
c. dy/dx = 2(3x-5).
d. dy/dx = 6(3x-5).
e. dy/dx = 6x.

(5) Suppose $y = x^{-1/2}$. Then the derivative of y with respect to x is given by
a. dy/dx = x/2.
b. dy/dx = $x^{-3/2}$.
c. dy/dx = $- x^{-1/2}$.
d. dy/dx = $(-1/2)\, x^{-3/2}$.
e. none of the above.

(6) Suppose $y = 2\,(3+4x)^5$. Then the derivative of y with respect to x is
a. dy/dx = 8 .
b. dy/dx = 120 .
c. dy/dx = $10\,(3+4x)^4$.
d. dy/dx = $(3+4x)^4$.
e. dy/dx = $40\,(3+4x)^4$.

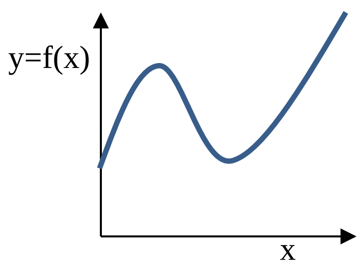
(7) Which of the following functions has constant slope (or derivative)?

a. $y = \ln(2x)$.
b. $y = \exp(3x)$.
c. $y = 5 + 2x$.
d. $y = 5 + (4/x)$.
e. $y = 5 + 4x + (3/2)x^2$.
f. $y = 5x^{-2}$.

(8) If x increases by 2 percent, then ln(x) increases by about

a. 0.02 percent.
b. 0.02 units.
c. 2 percent.
d. 2 units.
e. ln(2), or about 0.693 units.

The next question refers to the following graph of $y = f(x)$.



(9) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at

a. no point on the graph.
b. one point on the graph.
c. two points on the graph.
d. three points on the graph.
e. four points on the graph.
f. more than four points on the graph.

(10) Suppose we wish to maximize the function $y = f(x)$, which is continuously differentiable. Assuming there are no restrictions on the possible values of x, the maximizing value x^* must satisfy

a. $x^* = 0$.
b. $f(x^*) = 0$.
c. $df/dx = 0$, if $x = x^*$.
d. $d^2f/dx^2 = 0$, if $x = x^*$.
e. All of the above.

(11) If y is proportional to x, then the elasticity of y with respect to x equals

a. zero.
b. one-half.
c. one.
d. two.
e. x.
f. cannot be determined from information given.

(12) Consider the following functions. Which has constant elasticity?

a. $y = \ln(2x)$.
b. $y = \exp(3x)$.
c. $y = 5 + 2x$.
d. $y = 5 + (4/x)$.
e. $y = 5 + 4x + (3/2)x^2$.
f. $y = 5x^{-2}$.

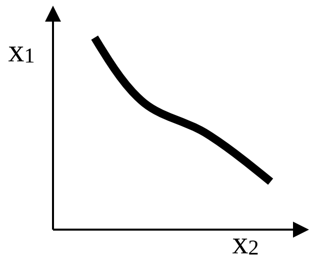
(13) A straight line has constant

a. slope.
b. elasticity.
c. both of the above.
d. none of the above.

(14) Suppose y depends on both x_1 and x_2, so that $y = f(x_1, x_2)$. By definition, the partial derivative of y with respect to x_1 is the ratio of the change in y to the change in x_1 when x_2

a. equals zero.
b. is held constant.
c. equals x_1.
d. changes so as to keep y constant.

The next question refers to the following graph of a level curve, or contour, of the function $y = f(x_1,x_2)$.



(15) According to this graph, if x_1 increases and y is to be held constant, then x_2 must

a. increase.
b. decrease.
c. remain constant.
d. equal zero.
e. cannot be determined from the information given.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals 2 at a particular value of x. Moreover, the elasticity of y with respect to x equals 0.6 . Further suppose that x increases by 3 *units*. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

b. By about how much?

units

(2) [4 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 2$, and $\partial y/\partial x_2 = 0.5$, and that the partial elasticities are $\varepsilon_1 = 3$ and $\varepsilon_2 = 0.25$. Further suppose that x_1 increases by 2 *percent* and simultaneously x_2 increases by 4 *percent*. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

b. By about how much?

percent

(3) [4 pts] GDP equals the number of workers times productivity. Suppose the number of workers increases by 1 percent and productivity increases by 3 percent.

a. Will GDP *increase* or *decrease*?

b. By about how much?

percent

(4) [4 pts] Per-capita income is defined as total income divided by population. Suppose total income increases by 5 percent and population increases by 2 percent.

a. Will per-capita income *increase* or *decrease*?

b. By about how much?

percent

(5) [8 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 2$, and $\partial y/\partial x_2 = 3$. First, suppose that x_1 decreases by 6 units but x_2 does not change.

a. Will y *increase* or *decrease*?

b. By about how much?

units

Now suppose that x_1 decreases by 6 units but we want y to remain constant. To keep y constant, we must change the value of x_2.

c. Must x_2 *increase* or *decrease*?

d. By about how much?

units

(6) [4 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 15$, and $\partial y/\partial x_2 = 3$. Now consider a graph of the level curve of this function, with x_1 on the vertical axis and x_2 on the horizontal axis.

a. Does the level curve of the function slope *up* or *down* at that point?

b. Give the slope of the level curve at this point.

III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = 3x^2 - 12x + 20$.

a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

b. Compute the value x^* that minimizes this function.

c. For what range of values of x does this function slope up? For what range of values does it slope down?

d. Find the minimum value, y^*, of the function itself.

(2) [Partial elasticities: 6 pts] Suppose $y = x_1^2 (x_2 - 5)^2$.

a. Find an expression for ε_1, the partial elasticity of y with respect to x_1. The variable y should *not* appear in your answer.

b. Find an expression for ε_2, the partial elasticity of y with respect to x_2. The variable y should *not* appear in your answer.

(3) [MRS: 12 pts] Suppose $y = f(x_1,x_2) = 4\,x_1^{0.5} + 6\,x_2^{0.5}$. The arguments x_1 and x_2 are strictly positive.

a. Find an expression for the partial derivative of y with respect to x_1 .

b. Find an expression for the partial derivative of y with respect to x_2 .

c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

(4) [MRS: 12 pts] Suppose $y = f(x_1,x_2) = (x_1 - 5)^2 (x_2 + 2)^3$. The arguments x_1 and x_2 are strictly positive.

a. Find an expression for the partial derivative of y with respect to x_1 .

b. Find an expression for the partial derivative of y with respect to x_2 .

c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

IV. CRITICAL THINKING: [4 pts] Answer *one* question below (your choice). Circle the question you are answering. Justify your answer and show your work.

(1) Let the function $f(x)$ represent net benefit to society as a function of some variable x, which might represent the amount of output of some good or service. Now $f(x)$ is itself the difference of two other functions $b(x)$ and $c(x)$, representing benefits and costs of x, respectively, so that $f(x) = b(x) - c(x)$. When $f(x)$ is maximized, what must be the relationship between the derivatives db/dx and dc/dx ?

(2) Suppose $P \times Q = a$, where a is some constant, for all values of P and Q. Find the elasticity of Q with respect to P. Does the elasticity depend on the value of a ?

[end of exam]