EXAMINATION #2 ANSWER KEY
"Consumers and Demand"

Version A

I. Multiple choice
(1) b .   (2) c.   (3) c.   (4) d.

II. Short answer
(1)   In this question, we must try to determine where points A, B, and C lie in relation to the indifference curve that passes through points X and Y. The assumption of monotonicity implies that this indifference curve must slope down. More importantly for this question, the assumption of diminishing marginal rate of substitution implies that this indifference curve is curved (not straight) although we do not know *how* curved it is.
a. more preferred, because it lies on the line connecting X and Y, and diminishing MRS implies that "averages are preferred to extremes."
b. cannot be determined, because it lies below the line connecting X and Y.
c. more preferred because it lies above the line connecting X and Y.

(2)   a. elastic.   b. decrease.   c. 6%.
d. decrease.   e. 1%.

(3)   a. luxury or superior good.   b. increase.   c. 7 %.
d. increase.   e. 2 %.

(4)   Note: This graph is based on Hal Varian's presentation of income and substitution effects in his intermediate microeconomics textbooks. Other textbooks offer different graphical presentations.
a. $4.   b. 9 units.   c. $12.
d. 4 units.   e. -2 units.   f. -3 units.

(5)   a. $\varepsilon^{\text{comp}} = -0.25$.   b. decrease.   c. 3 %.
d. 0.5 %.   e. 2.5 %.

(6)   a. Laspeyres = 140.   b. Paasche = 130.   c. $\sqrt{140 \times 130}$.

III. Problems
(1)   [Budgets and choice]
a. Equation for budget line: $1 q_1 + 2 q_2 = 34$.
b. MRS $= MU_2/MU_1 = (3 q_1^{1/2}) / (2 q_2^{1/2})$.
c. Solve MRS $= p_2/p_1 = 2/1$ jointly with equation for budget line to get $q_1^* = 16$, $q_2^* = 9$.

(2)   [Properties of individual demand functions]
a. Check homogeneity of the demand function:
$$\frac{(a\ I) - 5(a p_2)}{2(a\ p_1)} = \frac{a\ (I - 5p_2)}{a(2p_1)} = \frac{(I - 5p_2)}{(2p_1)} .$$

Note that the "a" factor cancels. So multiplying income and prices by some arbitrary positive factor a *does not* change the quantity demanded. The function is <u>homogeneous of degree zero</u> in income and prices.

b. $\frac{\partial q_1^*}{\partial p_1} = -\frac{(I-5p_2)}{2}p_1^{-2}$. This expression is negative, so good #1 is an <u>ordinary good</u>, not a Giffen good.

c. $\frac{\partial q_1^*}{\partial I} = \frac{1}{2\,p_1}$. This expression is positive, so good #1 is a <u>normal good</u>, not an inferior good.

d. $\frac{\partial q_1^*}{\partial p_2} = -\frac{5}{2\,p_1}$. This expression is negative, so goods #1 and #2 are <u>complements</u>, not substitutes.

(3) [Finding individual demand functions]
a. MRS = $MU_2/MU_1 = (3q_1) / q_2$.
Solve MRS = p_2/p_1 jointly with $I = p_1 q_1 + p_2 q_2$ to get
b. $q_1^* = \frac{I}{4\,p_1}$, and   c. $q_2^* = \frac{3I}{4\,p_2}$.

III. Critical thinking

(1) In most situations involving consumer choice, a Laspeyres cost of living index increases fastest (due to substitution bias), a Paasche COL index increases slowest, and a Fisher COL index lies in between. However, in this special situation, all three COL indices increase at exactly the *same rate*. Here is a proof with algebra:

Laspeyres COL index
$$= \frac{p_c^{new}q_c^{old}+p_d^{new}q_d^{old}}{p_c^{old}q_c^{old}+p_d^{old}q_d^{old}} \times 100 = \frac{1.10\,p_c^{old}q_c^{old}+1.10\,p_d^{old}q_d^{old}}{p_c^{old}q_c^{old}+p_d^{old}q_d^{old}} \times 100 = 1.10 \times 100 = 110 \ .$$

Paasche COL index
$$= \frac{p_c^{new}q_c^{new}+p_d^{new}q_d^{new}}{p_c^{old}q_c^{new}+p_d^{old}q_d^{new}} \times 100 = \frac{1.10\,p_c^{old}q_c^{new}+1.10\,p_d^{old}q_d^{new}}{p_c^{old}q_c^{new}+p_d^{old}q_d^{new}} \times 100 = 1.10 \times 100 = 110 \ .$$

Fisher COL index $= \sqrt{Laspeyres \times Paasche} = \sqrt{110 \times 110} = 110 \ .$

(2) The amount the consumer would be willing to pay for the price decrease is the *gain in consumer surplus*, which equals the area of rectangle A plus the area of triangle B in the graph below. The area of rectangle A is $(3-1) \times 20 = \$40$. The area of triangle B is unknown, but obviously positive, so the consumer would be willing to pay some amount greater than $40 for the price reduction. Thus the consumer would *prefer a reduction in price from $3 to $1* to an increase in income of $40. [Full credit requires a graph like that below.]
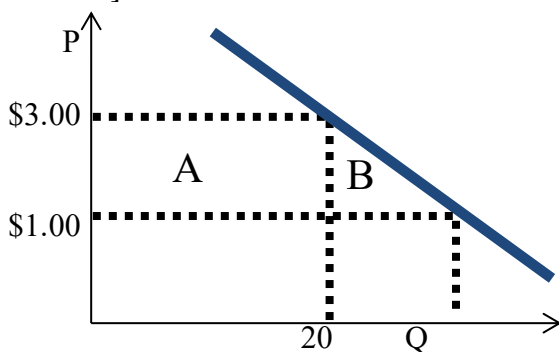
Version B

I. Multiple choice
(1) c .   (2) a.   (3) c.   (4) b.

II. Short answer
(1)   In this question, we must try to determine where points A, B, and C lie in relation to the indifference curve that passes through points X and Y. The assumption of monotonicity implies that this indifference curve must slope down. More importantly for this question, the assumption of diminishing marginal rate of substitution implies that this indifference curve is curved (not straight) although we do not know *how* curved it is.
a. cannot be determined, because it lies below the line connecting X and Y.
b. more preferred because it lies above the line connecting X and Y.
c. more preferred, because it lies on the line connecting X and Y, and diminishing MRS implies that "averages are preferred to extremes."

(2)   a. inelastic.   b. increase.   c. 4%.
         d. decrease.   e. 6%.

(3)   a. necessary good.   b. increase.   c. 4 %.
         d. decrease.   e. 1 %.

(4)   Note: This graph is based on Hal Varian's presentation of income and substitution effects in his intermediate microeconomics textbooks. Other textbooks offer different graphical presentations.
         a. $10.   b. 3 units.   c. $3.
         d. 12 units.   e. 6 units.   f. 3 units.

(5)   a. $\varepsilon^{\text{comp}} = -1.3$.   b. increase.   c. 14 %.
         d. 1 %.   e. 13 %.

(6)   a. Laspeyres = 160.   b. Paasche = 140.   c. $\sqrt{160 \times 140}$.

III. Problems
(1)   [Budgets and choice]
a. Equation for budget line: $1 q_1 + 4 q_2 = 30$.
b. $MRS = MU_2/MU_1 = q_1^2 / q_2^2$.
c. Solve $MRS = p_2/p_1 = 4/1$ jointly with equation for budget line to get $q_1^* = 10$, $q_2^* = 5$.

(2)   [Properties of individual demand functions]
a. Check homogeneity of the demand function:
$$0.2 \, (a \, p_1)^{-0.4} \, (a \, I)^{-0.1} \, (a \, p_2)^{0.3} = (a^{-0.4} a^{-0.1} a^{0.3})(47 \, I^{0.9} p_1^{-0.8} p_2^{-0.1})$$
$$= (a^{-0.2})(47 \, I^{0.9} p_1^{-0.8} p_2^{-0.1}) \ .$$
Note that the "a" factor does NOT cancel. So multiplying income and prices by some arbitrary positive factor a *changes* the quantity demanded. The function is <u>NOT homogeneous of degree zero</u> in income and prices. (Therefore is not a legitimate demand function, but we proceed anyway.)
b. $\frac{\partial q_1^*}{\partial p_1} = -0.08 \, p_1^{-1.4} \, I^{-0.1} \, p_2^{0.3}$. This expression is negative, so good #1 is an <u>ordinary good</u>, not a Giffen good.

c. $\frac{\partial q_1^*}{\partial I} = -0.02\, p_1^{-0.4}\, I^{-1.1}\, p_2^{0.3}$. This expression is negative, so good #1 is an <u>inferior good</u>, not a normal good.

d. $\frac{\partial q_1^*}{\partial p_2} = 0.06\, p_1^{-0.4}\, I^{-0.1}\, p_2^{-0.7}$. This expression is positive, so goods #1 and #2 are <u>substitutes</u>, not complements.

(3)   [Finding individual demand functions]

a. MRS = $MU_2/MU_1 = q_1 / (4q_2)$.

Solve MRS = p_2/p_1 jointly with $I = p_1 q_1 + p_2 q_2$ to get

b. $q_1^* = \dfrac{4\,I}{5\,p_1}$,   and   c. $q_2^* = \dfrac{I}{5\,p_2}$.

III. Critical thinking
Same as Version A.

[end of answer key]