

EXAMINATION #1 ANSWER KEY “Mathematical Tools”

Version A

I. Multiple choice

(1)a. (2)d. (3)b. (4)c. (5)a. (6)b. (7)e. (8)d. (9)a. (10)b.

II. Short answer

- (1) a. increase b. 6 units, using derivative since change is given in units.
 (2) a. increase b. 6 %, using elasticities since changes are given in percent.
 (3) a. increase b. 2 %, using approximation rule for products.
 (4) a. increase b. 3 %, using approximation rule for ratios.
 (5) a. increase b. 12 units c. decrease d. 6 units.
 (6) a. down b. $-0.5 = \frac{\partial y / \partial x_2}{\partial y / \partial x_1}$.

III. Problems

- (1) a. $dy/dx = -2x+10$. b. $x^* = 5$.
 c. The function slopes up if $dy/dx = -2x+10 > 0$, which implies $x < 5$.
 The function slopes down if $dy/dx = -2x+10 < 0$, which implies $x > 5$.
 d. $y^* = f(x^*) = f(5) = 20$.
- (2) a. $\epsilon_1 = \frac{x_1}{2(x_1-6)}$. b. $\epsilon_2 = 2$.
- (3) a. $\frac{\partial y}{\partial x_1} = 2 x_1^{-2}$ b. $\frac{\partial y}{\partial x_2} = 3 x_2^{-2}$ c. $MRS = \frac{\partial y / \partial x_2}{\partial y / \partial x_1} = \frac{3 x_1^2}{2 x_2^2} = \frac{3}{2} \left(\frac{x_1}{x_2} \right)^2$.
- (4) a. $\frac{\partial y}{\partial x_1} = 3(x_1 + 2)^2 (x_2 - 3)^2$ b. $\frac{\partial y}{\partial x_2} = (x_1 + 2)^3 2(x_2 - 3)$
 c. $MRS = \frac{\partial y / \partial x_2}{\partial y / \partial x_1} = \frac{2(x_1+2)}{3(x_2-3)}$.

IV. Critical thinking

Using the total derivative in elasticity form,
 $\% \text{ chg } y = 5\% \epsilon_1 + 5\% \epsilon_2 + 5\% \epsilon_3$
 $= 5\% (\epsilon_1 + \epsilon_2 + \epsilon_3)$
 $= 5\% (1) = 5\%$.

So y increases by 5%.

Version B

I. Multiple choice

(1)b. (2)a. (3)c. (4)a. (5)b. (6)d. (7)e. (8)c. (9)b. (10)a.

II. Short answer

- (1) a. increase b. 2 %, using elasticity since change is given in percent.
(2) a. increase b. 5 units, using derivative since changes are given in units.
(3) a. decrease b. 5 %, using approximation rule for products.
(4) a. increase b. 1 %, using approximation rule for ratios.
(5) a. increase b. 18 units c. decrease d. 9 units.
(6) a. down b. -3.

III. Problems

- (1) a. $dy/dx = 2x-8$. b. $x^* = 4$.
c. The function slopes up if $dy/dx = 2x-8 > 0$, which implies $x > 4$.
The function slopes up if $dy/dx = 2x-8 < 0$, which implies $x < 4$.
d. $y^* = f(x^*) = f(4) = 14$.
- (2) a. $\epsilon_1 = 1/3$. b. $\epsilon_2 = \frac{3x_2}{x_2+2}$.
- (3) a. $\frac{\partial y}{\partial x_1} = 3(x_1 - 5)^2(x_2 + 3)^4$ b. $\frac{\partial y}{\partial x_2} = (x_1 - 5)^3 4(x_2 + 3)^3$
c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{4(x_1-5)}{3(x_2+3)}$.
- (4) a. $\frac{\partial y}{\partial x_1} = 2x_1^{-0.5}$ b. $\frac{\partial y}{\partial x_2} = 3x_2^{-0.5}$ c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{3x_2^{0.5}}{2x_1^{0.5}} = \frac{3}{2} \left(\frac{x_1}{x_2}\right)^{0.5}$.

IV. Critical thinking

(Same as version A.)

[end of answer key]