

EXAMINATION #1 VERSION A
“Mathematical Tools”
September 5, 2017

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [3 pts each—30 pts total]

(1) Suppose the derivative of the function $y = f(x)$ equals 0.9 at a particular value of x . At that point, the graph of the function is

- upward-sloping.
- downward-sloping.
- vertical.
- horizontal.
- cannot be determined from the information given.

(2) Which of the following functions has constant slope (or derivative)?

- $y = 4x^{-3}$.
- $y = \ln(3x)$.
- $y = \exp(2x)$.
- $y = 2 + 3x$.
- $y = 7 + (6/x)$.
- $y = 2 + 6x + (1/3)x^3$.

(3) Suppose we wish to maximize the function $y = f(x)$, which is continuously differentiable. Assuming there are no restrictions on the possible values of x , the maximizing value x^* must satisfy

- $f(x^*) = 0$.
- $df/dx = 0$, if $x = x^*$.
- $d^2f/dx^2 = 0$, if $x = x^*$.
- $x^* = 0$.
- All of the above.

(4) If y is proportional to x , then the elasticity of y with respect to x equals

- zero.
- one-half.
- one.
- two.
- x .
- cannot be determined from information given.

(5) A straight line has constant

- slope.
- elasticity.
- both of the above.
- none of the above.

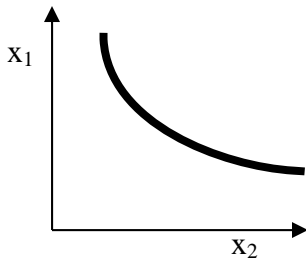
(6) Suppose y depends on both x_1 and x_2 , so that $y = f(x_1, x_2)$. By definition, the partial derivative of y with respect to x_1 is the ratio of the change in y to the change in x_1 when x_2

- equals zero.
- is held constant.
- equals x_1 .
- changes so as to keep y constant.

(7) Which of the following functions has constant partial *elasticities* (ϵ_1 and ϵ_2) ?

- a. $y = 5 + 2x_1 + 3x_2$.
- b. $y = 5x_1 + 2x_2 + 3(x_1x_2)^{1/2}$.
- c. $y = 5 + 2x_1^{-1} + 3x_2^{-1}$.
- d. $y = 5 + 2x_1^{1/2} + 3x_2^{1/2}$.
- e. $y = 5x_1^2x_2^3$.
- f. $y = 5(x_1+2)^2(x_2+3)^3$.

The next three questions refer to the following graph of a level curve, or contour, of the function $y = f(x_1, x_2)$.



(8) By definition, all points along the curve in this graph have identical values of

- a. x_1 .
- b. x_2 .
- c. the marginal rate of substitution.
- d. y .
- e. all of the above.
- f. none of the above.

(9) According to this graph, if x_2 decreases and y is to be held constant, then x_1 must

- a. increase.
- b. decrease.
- c. remain constant.
- d. be set equal to zero.
- e. cannot be determined from the information given.

(10) Along this level curve, as we move down and to the right, the marginal rate of substitution of x_2 for x_1 (that is, the $|\text{slope}|$ of the level curve with x_1 on the vertical axis and x_2 on the horizontal axis) is

- a. increasing.
- b. diminishing.
- c. infinite.
- d. constant and equal to zero.
- e. constant and equal to one.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals 3 at a particular value of x . Moreover, the elasticity of y with respect to x equals 0.5. Further suppose that x increases by 2 units. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y increase or decrease?

b. By about how much?

units

(2) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 0.5$, and $\partial y / \partial x_2 = 3$, and that the partial elasticities are $\epsilon_1 = 2$ and $\epsilon_2 = 0.5$. Further suppose that x_1 increases by 2 percent and simultaneously x_2 increases by 4 percent. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y increase or decrease?

b. By about how much?

percent

(3) [4 pts] Revenue equals price times quantity sold. Suppose price decreases by 3 percent and the quantity sold increases by 5 percent.

a. Will revenue increase or decrease?

b. By about how much?

%

(4) [4 pts] Average family income is defined as total income divided by the number of families. Suppose the total income increases by 5 percent and the number of families increases by 2 percent.

a. Will average family income increase or decrease?

b. By about how much?

%

(5) [8 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 2$, and $\partial y / \partial x_2 = 3$. First, suppose that x_2 increases by 4 units but x_1 does not change.

a. Will y *increase* or *decrease*?

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b. By about how much?

units

Now suppose that x_2 increases by 4 units but we want y to remain constant. To keep y constant, we must change the value of x_1 .

c. Must x_1 *increase* or *decrease*?

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d. By about how much?

units

(6) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 6$, and $\partial y / \partial x_2 = 3$. Now consider a graph of the level curve of this function, with x_1 on the vertical axis and x_2 on the horizontal axis.

a. Does the level curve of the function slope *up* or *down* at that point?

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b. Give the slope of the level curve at this point.

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III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = -x^2 + 10x - 5$.

- a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

- b. Compute the value x^* that maximizes this function.

- c. For what range of values of x does this function slope up? For what range of values does it slope down?

- d. Find the maximum value, y^* , of the function itself.

(2) [Partial elasticities: 6 pts] Suppose $y = (x_1 - 6)^{1/2} x_2^2$.

- a. Find an expression for ε_1 , the partial elasticity of y with respect to x_1 . The variable y should *not* appear in your answer.

- b. Find an expression for ε_2 , the partial elasticity of y with respect to x_2 . The variable y should *not* appear in your answer.

(3) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = -2x_1^{-1} - 3x_2^{-1}$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the partial derivative of y with respect to x_1 .

- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

(4) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = (x_1 + 2)^3 (x_2 - 3)^2$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the partial derivative of y with respect to x_1 .

- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

IV. CRITICAL THINKING: [4 pts]

Suppose $y = f(x_1, x_2, x_3)$. Let ε_1 denote the partial elasticity of y with respect to x_1 . Let ε_2 denote the partial elasticity of y with respect to x_2 . Let ε_3 denote the partial elasticity of y with respect to x_3 . Suppose $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 1$. If x_1 , x_2 , and x_3 all simultaneously increase by 5 percent, does y *increase* or *decrease*? By how about much? Justify your answer.

[end of exam]