

EXAMINATION #1 VERSION C
“Mathematical Tools”
September 6, 2016

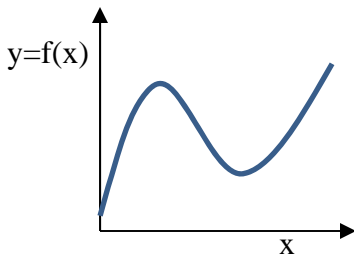
INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [3 pts each—30 pts total]

(1) Which of the following functions has constant slope (or derivative)?

- a. $y = \ln(5x)$.
- b. $y = \exp(5x)$.
- c. $y = 5 + 3x$.
- d. $y = 5 + (3/x)$.
- e. $y = 5 + 3x + (1/2)x^2$.
- f. $y = 3x^{-2}$.

The next question refers to the following graph of $y = f(x)$.



(2) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at

- a. no point on the graph.
- b. one point on the graph.
- c. two points on the graph.
- d. three points on the graph.
- e. four points on the graph.
- f. more than four points on the graph.

(3) Suppose y and x are strictly positive variables. If the derivative dy/dx is *negative*, then the elasticity of y with respect to x

- a. can be positive, negative, or zero.
- b. can be positive or negative but not zero.
- c. must be zero.
- d. must be positive.
- e. must be negative.

(4) Consider the following functions. Which has constant elasticity?

- a. $y = \ln(5x)$.
- b. $y = \exp(5x)$.
- c. $y = 5 + 3x$.
- d. $y = 5 + (3/x)$.
- e. $y = 5 + 3x + (1/2)x^2$.
- f. $y = 3x^{-2}$.

(5) Suppose y denotes the quantity demanded of apples (in pounds) and x denotes the price of apples (in dollars). Consider the demand function $y = f(x)$. The units of measure for the *elasticity* of y with respect to x are

- a. dollars.
- b. pounds.
- c. dollars per pound.
- d. pounds per dollar.
- e. The elasticity is unit-free.

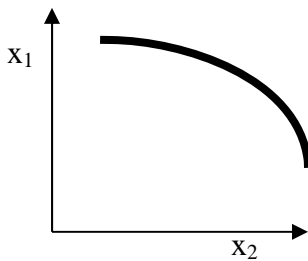
(6) Suppose y is inversely proportional to x , so that $y = a/x$, where a is a positive constant. Then the elasticity of y with respect to x equals

- a. zero.
- b. one-half.
- c. one.
- d. negative one.
- e. x .
- f. a .

(7) Which of the following functions has constant partial *elasticities* (ϵ_1 and ϵ_2) ?

- a. $y = 4 x_1^5 x_2^3$.
- b. $y = 4 (x_1+5)^5 (x_2+3)^3$.
- c. $y = 4 + 5 x_1 + 3 x_2$.
- d. $y = 5x_1 + 3x_2 + 4 (x_1x_2)^{1/2}$.
- e. $y = 4 + 5 x_1^{-1} + 3 x_2^{-1}$.
- f. $y = 4 + 5 x_1^{1/2} + 3 x_2^{1/2}$.

The next three questions refer to the following graph of a level curve, or contour, of the function $y = f(x_1, x_2)$.



(8) By definition, all points along the curve in this graph have identical values of

- a. x_1 .
- b. x_2 .
- c. the marginal rate of substitution.
- d. y .
- e. all of the above.
- f. none of the above.

(9) According to this graph, if x_2 increases and y is to be held constant, then x_1 must

- a. be set equal to zero.
- b. remain constant.
- c. increase.
- d. decrease.
- e. cannot be determined from the information given.

(10) Along this level curve, as we move down and to the right, the marginal rate of substitution of x_2 for x_1 (that is, the $|\text{slope}|$ of the level curve with x_1 on the vertical axis and x_2 on the horizontal axis) is

- a. increasing.
- b. diminishing.
- c. infinite.
- d. constant and equal to zero.
- e. constant and equal to one.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals 0.5 at a particular value of x . Moreover, the elasticity of y with respect to x equals 0.2. Further suppose that x increases by 4 units. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y increase or decrease?

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b. By about how much?

units

(2) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 5$, and $\partial y / \partial x_2 = 2$, and that the partial elasticities are $\epsilon_1 = 1.2$ and $\epsilon_2 = 0.4$. Further suppose that x_1 increases by 2 percent and simultaneously x_2 increases by 4 percent. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y increase or decrease?

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b. By about how much?

percent

(3) [4 pts] Revenue equals price times quantity sold. Suppose price increases by 6 percent and the quantity sold decreases by 3 percent.

a. Will revenue increase or decrease?

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b. By about how much?

%

(4) [4 pts] The average product of labor equals total output divided by total labor input. Suppose total output in the construction sector increases by 7 percent and total labor input increases by 2 percent.

a. Will the average product increase or decrease?

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b. By about how much?

%

(5) [8 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 2$, and $\partial y / \partial x_2 = 4$. First, suppose that x_2 increases by 5 units but x_1 does not change.

a. Will y *increase* or *decrease*?

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b. By about how much?

units

Now suppose that x_2 increases by 5 units but we want y to remain constant. To keep y constant, we much change the value of x_1 .

c. Must x_1 *increase* or *decrease*?

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d. By about how much?

units

(6) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 2$, and $\partial y / \partial x_2 = 6$. Now consider a graph of the level curve of this function, with x_1 on the vertical axis and x_2 on the horizontal axis.

a. Does the level curve of the function slope *up* or *down* at that point?

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b. Give the slope of the level curve at this point.

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III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = -x^2 + 10x - 15$.

- a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

- b. Compute the value x^* that maximizes this function.

- c. For what range of values of x does this function slope up? For what range of values does it slope down?

- d. Find the maximum value, y^* , of the function itself.

(2) [Partial elasticities: 6 pts] Suppose $y = (x_1 + 4)^2 x_2^4$.

- a. Find an expression for ε_1 , the partial elasticity of y with respect to x_1 . The variable y should *not* appear in your answer.

- b. Find an expression for ε_2 , the partial elasticity of y with respect to x_2 . The variable y should *not* appear in your answer.

(3) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = (-3/x_1) + (-2/x_2)$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the partial derivative of y with respect to x_1 .

- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the $|\text{slope}|$ of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

(4) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = (x_1 + 5)^3 (x_2 - 6)^2$. The arguments x_1 and x_2 are strictly positive.

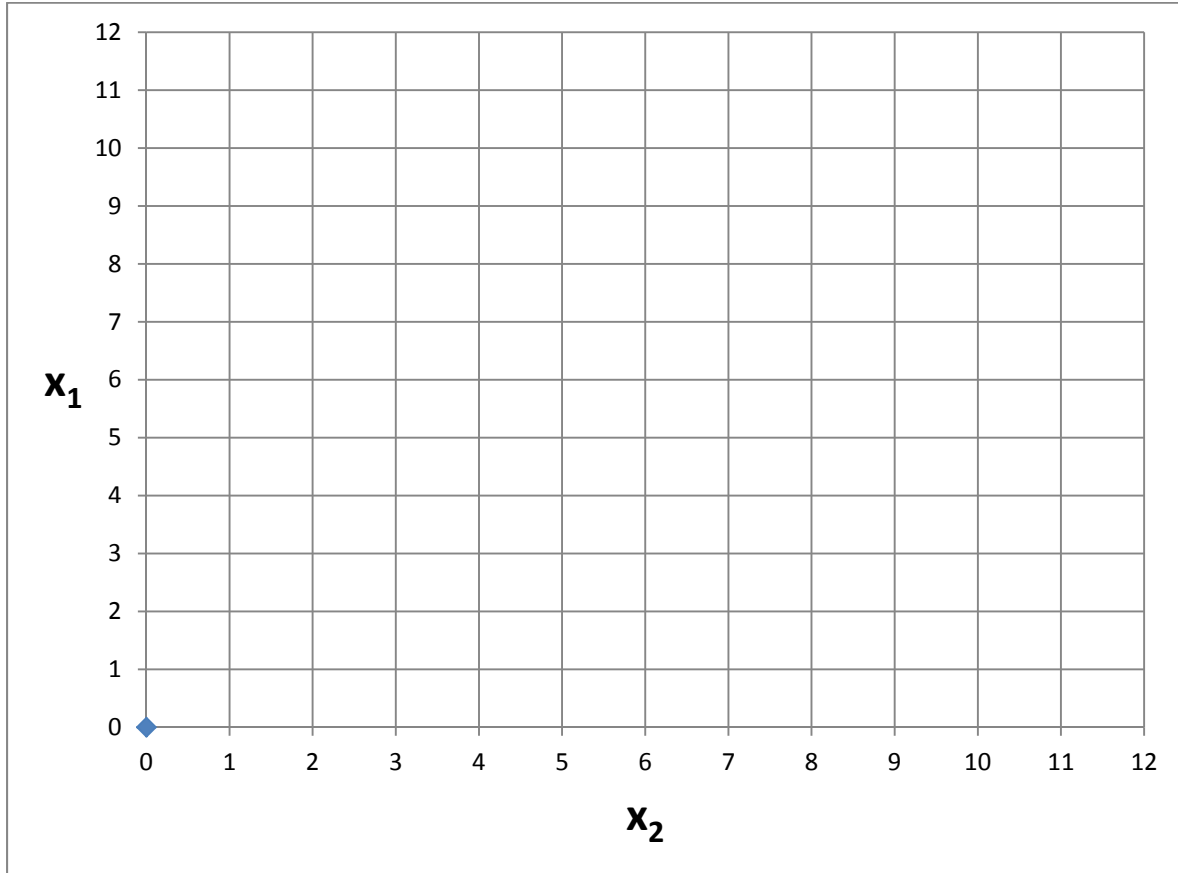
- a. Find an expression for the partial derivative of y with respect to x_1 .

- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

IV. CRITICAL THINKING: [4 pts]

Suppose $y = f(x_1, x_2)$. Further suppose $\partial y / \partial x_1$ is positive but $\partial y / \partial x_2$ is always zero. Sketch two level curves of this function in the graph below.



[end of exam]