

EXAMINATION #1 VERSION B
“Mathematical Tools”
September 6, 2016

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [3 pts each—30 pts total]

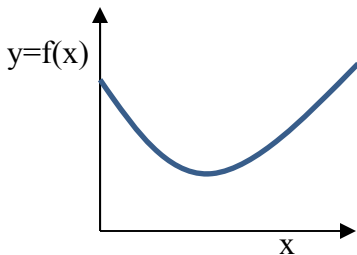
(1) Which of the following functions has constant slope (or derivative)?

- a. $y = 4x^{-2}$.
- b. $y = \ln(3x)$.
- c. $y = \exp(3x)$.
- d. $y = 5 + 2x$.
- e. $y = 4 + (2/x)$.
- f. $y = 5 + 2x + (1/2)x^2$.

(3) Suppose y and x are strictly positive variables. If the derivative dy/dx is *positive*, then the elasticity of y with respect to x

- a. must be zero.
- b. must be positive.
- c. must be negative.
- d. can be positive or negative but not zero.
- e. can be positive, negative, or zero.

The next question refers to the following graph of $y = f(x)$.



(2) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at

- a. no point on the graph.
- b. one point on the graph.
- c. two points on the graph.
- d. three points on the graph.
- e. four points on the graph.
- f. more than four points on the graph.

(4) Consider the following functions. Which has constant elasticity?

- a. $y = 4x^{-2}$.
- b. $y = \ln(3x)$.
- c. $y = \exp(3x)$.
- d. $y = 5 + 2x$.
- e. $y = 4 + (2/x)$.
- f. $y = 5 + 2x + (1/2)x^2$.

(5) Suppose y denotes the quantity demanded of electricity (in kilowatt-hours) and x denotes the price of electricity (in cents). Consider the demand function $y = f(x)$. The units of measure for the *derivative* of y with respect to x are

- a. cents.
- b. kilowatt-hours.
- c. cents per kilowatt-hour.
- d. kilowatt-hours per cent.
- e. The derivative is unit-free.

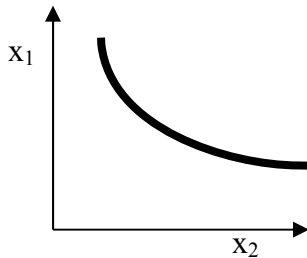
(3.5) If y does not depend on x , then the elasticity of y with respect to x equals

- a. zero.
- b. one-half.
- c. one.
- d. two.
- e. x .
- f. cannot be determined from information given.

(5.10) Which of the following functions has constant partial derivatives ($\partial y/\partial x_1$ and $\partial y/\partial x_2$) ?

- a. $y = 8 + 3x_1 + 4x_2$.
- b. $y = 3x_1 + 4x_2 + 6(x_1x_2)^{1/3}$.
- c. $y = 8 + 3x_1^{-1} + 4x_2^{-1}$.
- d. $y = 8 + 3x_1^{1/3} + 4x_2^{1/3}$.
- e. $y = 8x_1^3x_2^4$.
- f. $y = 8(x_1+3)^3(x_2+4)^4$.

The next three questions refer to the following graph of a level curve, or contour, of the function $y = f(x_1, x_2)$.



(6.5) By definition, all points along the curve in this graph have identical values of

- a. the marginal rate of substitution.
- b. y .
- c. x_1 .
- d. x_2 .
- e. all of the above.
- f. none of the above.

(6.6) According to this graph, if x_2 decreases and y is to be held constant, then x_1 must

- a. increase.
- b. decrease.
- c. remain constant.
- d. be set equal to zero.
- e. cannot be determined from the information given.

(6.7) Along this level curve, as we move down and to the right, the marginal rate of substitution of x_2 for x_1 (that is, the |slope| of the level curve with x_1 on the vertical axis and x_2 on the horizontal axis) is

- a. increasing.
- b. diminishing.
- c. infinite.
- d. constant and equal to zero.
- e. constant and equal to one.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals 0.5 at a particular value of x . Moreover, the elasticity of y with respect to x equals 0.2. Further suppose that x increases by 10 percent. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

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b. By about how much?

percent

(2) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 5$, and $\partial y / \partial x_2 = 2$, and that the partial elasticities are $\epsilon_1 = 1.2$ and $\epsilon_2 = 0.4$. Further suppose that x_1 increases by 3 units and simultaneously x_2 increases by 1 unit. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

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b. By about how much?

units

(3) [4 pts] Revenue equals price times quantity sold. Suppose price increases by 2 percent and the quantity sold decreases by 5 percent.

a. Will revenue *increase* or *decrease*?

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b. By about how much?

%

(4) [4 pts] The average product of labor equals total output divided by total labor input. Suppose total output in the construction sector increases by 1 percent and total labor input increases by 4 percent.

a. Will the average product *increase* or *decrease*?

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b. By about how much?

%

(5) [8 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 3$, and $\partial y / \partial x_2 = 4$. First, suppose that x_1 increases by 8 units but x_2 does not change.

a. Will y *increase* or *decrease*?

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b. By about how much?

units

Now suppose that x_1 increases by 8 units but we want y to remain constant. To keep y constant, we much change the value of x_2 .

c. Must x_2 *increase* or *decrease*?

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d. By about how much?

units

(6) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 10$, and $\partial y / \partial x_2 = 5$. Now consider a graph of the level curve of this function, with x_1 on the vertical axis and x_2 on the horizontal axis.

a. Does the level curve of the function slope *up* or *down* at that point?

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b. Give the slope of the level curve at this point.

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III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = -x^2 + 8x + 10$.

- a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

- b. Compute the value x^* that maximizes this function.

- c. For what range of values of x does this function slope up? For what range of values does it slope down?

- d. Find the maximum value, y^* , of the function itself.

(2) [Partial elasticities: 6 pts] Suppose $y = x_1^3 (x_2 + 2)^4$.

- a. Find an expression for ε_1 , the partial elasticity of y with respect to x_1 . The variable y should *not* appear in your answer.

- b. Find an expression for ε_2 , the partial elasticity of y with respect to x_2 . The variable y should *not* appear in your answer.

(3) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = -6x_1^{-0.5} - 4x_2^{-0.5}$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the partial derivative of y with respect to x_1 .

- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

(4) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = (x_1 + 4)^3 (x_2 - 5)^2$. The arguments x_1 and x_2 are strictly positive.

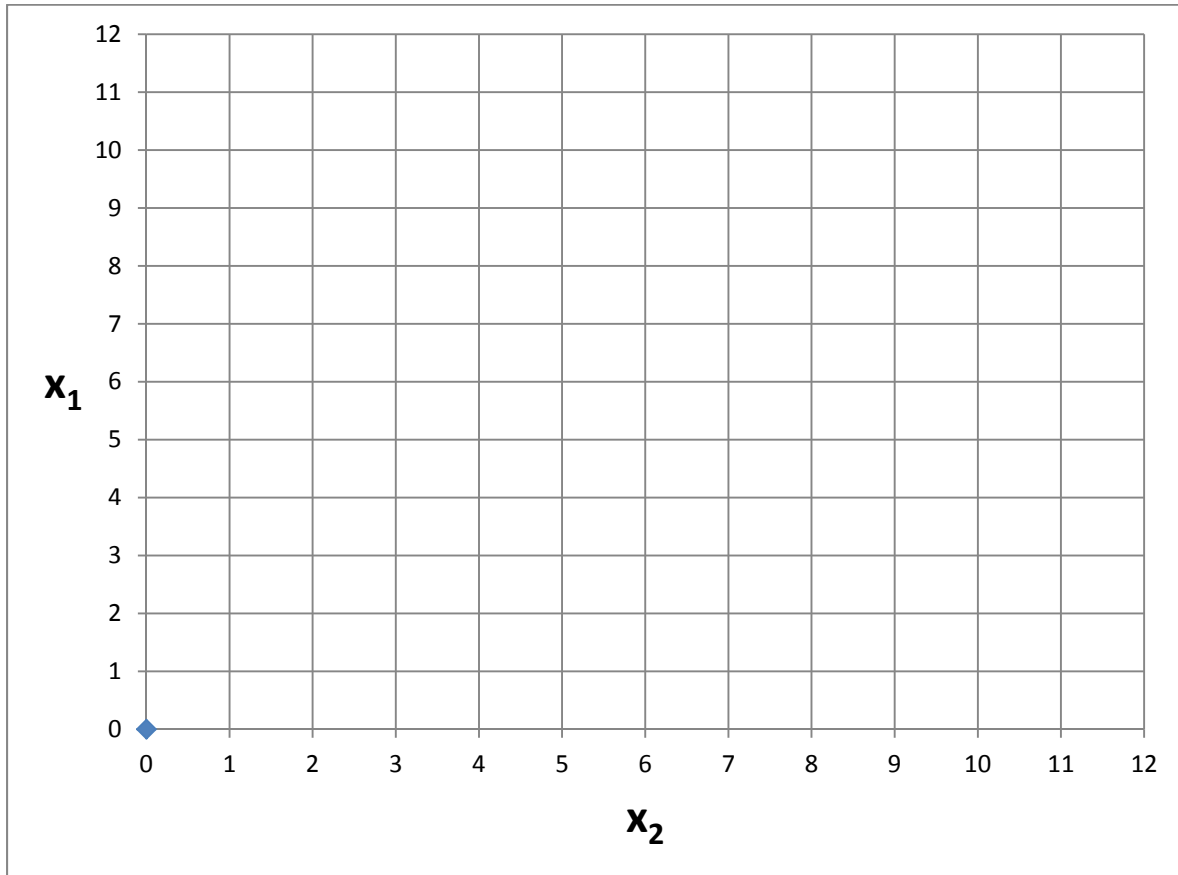
- a. Find an expression for the partial derivative of y with respect to x_1 .

- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

IV. CRITICAL THINKING: [4 pts]

Suppose $y = f(x_1, x_2)$. Further suppose $\partial y / \partial x_1$ is always half as large as $\partial y / \partial x_2$. Sketch two level curves of this function in the graph below.



[end of exam]