ECON 173 - Intermediate Microeconomic Analysis
Drake University, Fall 2016
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Signature: _____

Printed name: _____

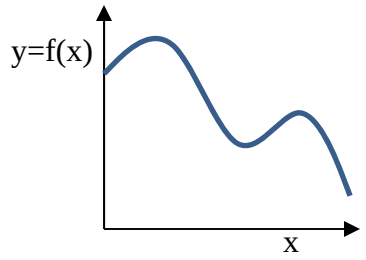
EXAMINATION #1 VERSION A
"Mathematical Tools"
September 6, 2016

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [3 pts each—30 pts total]

(1) Which of the following functions has constant slope (or derivative)?
a. $y = 3 + 2\,x$.
b. $y = 7 + (2/x)$.
c. $y = 3 + 2\,x + (1/2)\,x^2$.
d. $y = 4\,x^{-2}$.
e. $y = \ln(2x)$.
f. $y = \exp(3x)$.

The next question refers to the following graph of $y = f(x)$.



(2) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at
a. no point on the graph.
b. one point on the graph.
c. two points on the graph.
d. three points on the graph.
e. four points on the graph.
f. more than four points on the graph.

(3) Suppose y and x are strictly positive variables. If the derivative dy/dx is *negative*, then the elasticity of y with respect to x
a. must be zero.
b. must be positive.
c. must be negative.
d. can be positive or negative but not zero.
e. can be positive, negative, or zero.

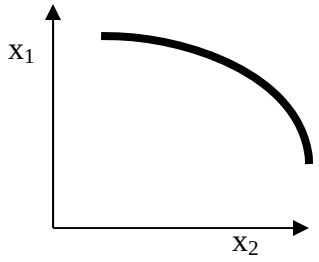
(4) Consider the following functions. Which has constant elasticity?
a. $y = 3 + 2\,x$.
b. $y = 7 + (2/x)$.
c. $y = 3 + 2\,x + (1/2)\,x^2$.
d. $y = 4\,x^{-2}$.
e. $y = \ln(2x)$.
f. $y = \exp(3x)$.

(5) Suppose y denotes the quantity demanded of electricity (in kilowatt-hours) and x denotes the price of electricity (in cents). Consider the demand function $y = f(x)$. The units of measure for the *elasticity* of y with respect to x are
a. cents.
b. kilowatt-hours.
c. cents per kilowatt-hour.
d. kilowatt-hours per cent.
e. The elasticity is unit-free.

(6) If y is proportional to x, then the elasticity of y with respect to x equals
a. zero.
b. one-half.
c. one.
d. two.
e. x.
f. cannot be determined from information given.

(7) Which of the following functions has constant partial *elasticities* (ε_1 and ε_2) ?
a. $y = 8 + 3\,x_1 + 4\,x_2$.
b. $y = 3x_1 + 4x_3 + 6\,(x_1x_2)^{1/3}$.
c. $y = 8 + 3\,x_1^{-1} + 4\,x_2^{-1}$.
d. $y = 8 + 3\,x_1^{1/3} + 4\,x_2^{1/3}$.
e. $y = 8\,x_1^{3}\,x_2^{4}$.
f. $y = 8\,(x_1+3)^3\,(x_2+4)^4$.

The next three questions refer to the following graph of a level curve, or contour, of the function $y = f(x_1,x_2)$.



(8) By definition, all points along the curve in this graph have identical values of
a. y .
b. x_1 .
c. x_2 .
d. the marginal rate of substitution.
e. all of the above.
f. none of the above.

(9) According to this graph, if x_2 increases and y is to be held constant, then x_1 must
a. increase.
b. decrease.
c. remain constant.
d. be set equal to zero.
e. cannot be determined from the information given.

(10) Along this level curve, as we move down and to the right, the marginal rate of substitution of x_2 for x_1 (that is, the |slope| of the level curve with x_1 on the vertical axis and x_2 on the horizontal axis) is
a. increasing.
b. diminishing.
c. infinite.
d. constant and equal to zero.
e. constant and equal to one.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals 0.5 at a particular value of x. Moreover, the elasticity of y with respect to x equals 0.2 . Further suppose that x increases by 10 *units*. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

b. By about how much? units

(2) [4 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 5$, and $\partial y/\partial x_2 = 2$, and that the partial elasticities are $\varepsilon_1 = 1.2$ and $\varepsilon_2 = 0.4$. Further suppose that x_1 increases by 3 *percent* and simultaneously x_2 increases by 1 *percent*. [Hint: Some of this information is extraneous and not needed to answer this question.]

a. Will y *increase* or *decrease*?

b. By about how much? percent

(3) [4 pts] Revenue equals price times quantity sold. Suppose price increases by 4 percent and the quantity sold decreases by 5 percent.

a. Will revenue *increase* or *decrease*?

b. By about how much? %

(4) [4 pts] The average product of labor equals total output divided by total labor input. Suppose total output in the construction sector increases by 10 percent and total labor input increases by 4 percent.

a. Will the average product *increase* or *decrease*?

b. By about how much? %

(5) [8 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 3$, and $\partial y/\partial x_2 = 4$. First, suppose that x_2 increases by 3 units but x_1 does not change.

a. Will y *increase* or *decrease*?

b. By about how much? units

Now suppose that x_2 increases by 3 units but we want y to remain constant. To keep y constant, we much change the value of x_1.

c. Must x_1 *increase* or *decrease*?

d. By about how much? units

(6) [4 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 2$, and $\partial y/\partial x_2 = 5$. Now consider a graph of the level curve of this function, with x_1 on the vertical axis and x_2 on the horizontal axis.

a. Does the level curve of the function slope *up* or *down* at that point?

b. Give the slope of the level curve at this point.

III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = -x^2 + 6x - 5$.

a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

b. Compute the value x* that maximizes this function.

c. For what range of values of x does this function slope up? For what range of values does it slope down?

d. Find the maximum value, y*, of the function itself.

(2) [Partial elasticities: 6 pts] Suppose $y = (x_1 - 5)^2 x_2^3$.

a. Find an expression for ε_1, the partial elasticity of y with respect to x_1. The variable y should *not* appear in your answer.

b. Find an expression for ε_2, the partial elasticity of y with respect to x_2. The variable y should *not* appear in your answer.

(3) [MRS: 12 pts] Suppose $y = f(x_1,x_2) = 4\,x_1^{0.5} + 2\,x_2^{0.5}$. The arguments x_1 and x_2 are strictly positive.

a. Find an expression for the partial derivative of y with respect to x_1.

b. Find an expression for the partial derivative of y with respect to x_2.

c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

(4) [MRS: 12 pts] Suppose $y = f(x_1,x_2) = (x_1 - 3)^2 (x_2 + 4)^3$. The arguments x_1 and x_2 are strictly positive.

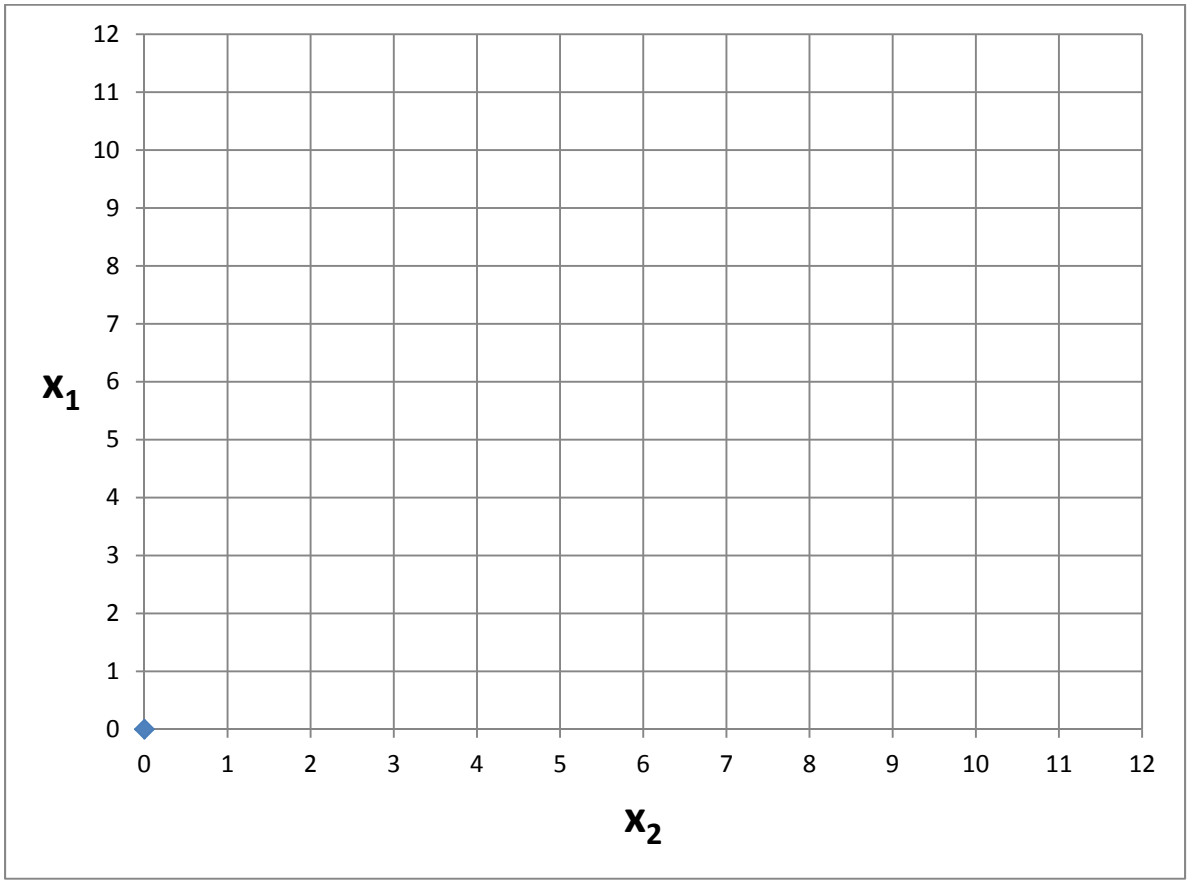
a. Find an expression for the partial derivative of y with respect to x_1.

b. Find an expression for the partial derivative of y with respect to x_2.

c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

IV. CRITICAL THINKING: [4 pts]

Suppose $y = f(x_1,x_2)$. Further suppose $\partial y/\partial x_1$ is always twice as large as $\partial y/\partial x_2$. Sketch two level curves of this function in the graph below.



[end of exam]