

## EXAMINATION #1 ANSWER KEY “Mathematical Tools”

### Version A

#### I. Multiple choice

(1)d. (2)c. (3)a. (4)a. (5)c. (6)d. (7)f. (8)c. (9)b. (10)b.

#### II. Short answer

- (1) a. increase                      b. 20 units, using derivative since change is given in units.  
(2) a. increase                      b. 5 %, using elasticities since changes are given in percent.  
(3) a. decrease                      b. 2 %, using approximation rule for products.  
(4) a. increase                      b. 3 %, using approximation rule for ratios.  
(5) a. increase                      b. 12 units                      c. decrease                      d. 6 units.  
(6) a. down                          b. -1.75.

#### III. Problems

- (1) a.  $dy/dx = -2x+10$ .      b.  $x^* = 5$ .  
c. Slopes up for  $x < 5$  because  $dy/dx$  is positive.  
Slopes down for  $x > 5$  because  $dy/dx$  is negative.  
d.  $y^* = f(x^*) = f(5) = 10$ .
- (2) a.  $\epsilon_1 = 2$ .                      b.  $\epsilon_2 = \frac{4x_2}{x_2-3}$
- (3) a.  $\frac{\partial y}{\partial x_1} = 7x_1^{-2}$                       b.  $\frac{\partial y}{\partial x_2} = 5x_2^{-2}$                       c.  $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{5x_1^2}{7x_2^2}$ .
- (4) a.  $\frac{\partial y}{\partial x_1} = 2(x_1 - 5)(x_2 - 4)^3$                       b.  $\frac{\partial y}{\partial x_2} = (x_1 - 5)^2 3(x_2 - 4)^2$   
c.  $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{3(x_1-5)}{2(x_2-4)}$ .

#### IV. Critical thinking

Using the total derivative in elasticity form,  
% chg in  $y$       =  $\epsilon_1$  (% chg  $x_1$ ) +  $\epsilon_2$  (% chg  $x_2$ )  
                         =  $\epsilon_1$  (3%) +  $\epsilon_2$  (3%)  
                         =  $(\epsilon_1 + \epsilon_2)$  (3%)  
                         = (1) (3%)      = 3%.

So  $y$  increases by about 3%.

## Version B

### I. Multiple choice

(1)f. (2)d. (3)c. (4)b. (5)d. (6)f. (7)b. (8)a. (9)b. (10)a.

### II. Short answer

- (1) a. increase                      b. 1.5 %, using elasticity since change is given in percent.  
(2) a. decrease                    b. 14 units, using derivative since changes are given in units.  
(3) a. increase                    b. 3 %, using approximation rule for products.  
(4) a. increase                    b. 4 %, using approximation rule for ratios.  
(5) a. increase                    b. 20 units                      c. decrease                      d. 10 units.  
(6) a. down                        b. -3.

### III. Problems

- (1) a.  $dy/dx = 2x - 20$ .            b.  $x^* = 10$ .  
c. Slopes up for  $x > 10$  because  $dy/dx$  is positive.  
Slopes down for  $x < 10$  because  $dy/dx$  is negative.  
d.  $y^* = f(x^*) = f(10) = -75$ .

- (2) a.  $\epsilon_2 = \frac{2x_1}{x_1 + 3}$ .                b.  $\epsilon_1 = 3$

- (3) a.  $\frac{\partial y}{\partial x_1} = 3x_1^{-3/2}$                 b.  $\frac{\partial y}{\partial x_2} = x_2^{-3/2}$                 c.  $MRS = \frac{\partial y / \partial x_2}{\partial y / \partial x_1} = \frac{1}{3} \left( \frac{x_1}{x_2} \right)^{3/2}$ .

- (4) a.  $\frac{\partial y}{\partial x_1} = 3(x_1 + 2)^3(x_2 + 1)^4$                 b.  $\frac{\partial y}{\partial x_2} = (x_1 + 2)^3 4(x_2 + 1)^3$   
c.  $MRS = \frac{\partial y / \partial x_2}{\partial y / \partial x_1} = \frac{4(x_1 + 2)}{3(x_2 + 1)}$ .

### IV. Critical thinking

Using the total derivative in elasticity form,  
% chg in  $y$     =  $\epsilon_1$  (% chg  $x_1$ ) +  $\epsilon_2$  (% chg  $x_2$ )  
                  =  $\epsilon_1$  (5%) +  $\epsilon_2$  (5%)  
                  =  $(\epsilon_1 + \epsilon_2)$  (5%)  
                  = (0) (5%)        = 0%.

So  $y$  remains constant (no increase or decrease).

## Version C

### I. Multiple choice

(1)b. (2)a. (3)e. (4)a. (5)d. (6)b. (7)d. (8)d. (9)d. (10)d.

### II. Short answer

- (1) a. increase                      b. 15 units, using derivative since change is given in units.  
(2) a. increase                      b. 0.8 %, using elasticities since changes are given in percent.  
(3) a. decrease                      b. 2 %, using approximation rule for products.  
(4) a. decrease                      b. 3 %, using approximation rule for ratios.  
(5) a. increase                      b. 18 units                      c. decrease                      d. 9 units.  
(6) a. down                              b. -5/3 .

### III. Problems

- (1) a.  $dy/dx = -20x+60$ .    b.  $x^* = 3$ .  
c. Slopes up for  $x < 3$  because  $dy/dx$  is positive.  
Slopes down for  $x > 3$  because  $dy/dx$  is negative.  
d.  $y^* = f(x^*) = f(3) = 80$ .
- (2) a.  $\epsilon_1 = 3$ .                      b.  $\epsilon_2 = \frac{2x_2}{x_2+4}$
- (3) a.  $\frac{\partial y}{\partial x_1} = x_1^{-1/2}$               b.  $\frac{\partial y}{\partial x_2} = 2 x_2^{-1/2}$               c.  $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = 2 \left(\frac{x_1}{x_2}\right)^{1/2}$ .
- (4) a.  $\frac{\partial y}{\partial x_1} = 5(x_1 - 3)^4(x_2 - 6)^2$               b.  $\frac{\partial y}{\partial x_2} = (x_1 - 3)^5 2(x_2 - 6)$   
c.  $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{2(x_1-3)}{5(x_2-6)}$ .

### IV. Critical thinking

Using the total derivative in elasticity form,  
% chg in  $y$     =  $\epsilon_1$  (% chg  $x_1$ ) +  $\epsilon_2$  (% chg  $x_2$ )  
                  =  $\epsilon_1$  (2%) +  $\epsilon_2$  (2%)  
                  =  $(\epsilon_1 + \epsilon_2)$  (2%)  
                  = (-1) (2%)    = - 2%.

So  $y$  decreases by about 2%.

[end of answer key]