EXAMINATION #1 VERSION A
"Mathematical Tools"
September 8, 2015

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets.

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [3 pts each—30 pts total]
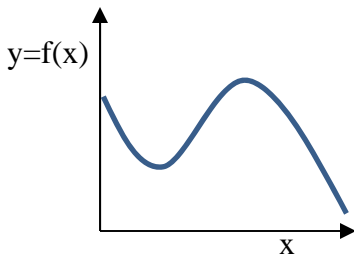
(1) Which of the following functions has constant slope (or derivative)?
a. $y = 4 x^{-3}$.
b. $y = \ln (3x)$.
c. $y = \exp (2x)$.
d. $y = 2 + 3 x$.
e. $y = 7 + (6/x)$.
f. $y = 2 + 6 x + (1/3) x^3$.

The next question refers to the following graph of $y = f(x)$.

(2) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at
a.   no point on the graph.
b.   one point on the graph.
c.   two points on the graph.
d.   three points on the graph.
e.   four points on the graph.
f.   more than four points on the graph.

(3) Consider the following functions. Which has constant elasticity?
a. $y = 4 x^{-3}$.
b. $y = \ln (3x)$.
c. $y = \exp (2x)$.
d. $y = 2 + 3 x$.
e. $y = 7 + (6/x)$.
f. $y = 2 + 6 x + (1/3) x^3$.

(4) A straight line has constant
a.   slope.
b.   elasticity.
c.   both of the above.
d.   none of the above.

(5) Suppose y denotes the quantity demanded of gasoline (in gallons) and x denotes the price of gasoline (in U.S. dollars). Consider the demand function y = f(x). The units of measure for the *derivative* of y with respect to x are
a.   gallons.
b.   dollars.
c.   gallons per dollar.
d.   The derivative is unit-free.

(6) Which of the following functions has constant partial *derivatives* ($\partial y/\partial x_1$ and $\partial y/\partial x_2$)?

a. $y = 8 (x_1-4)^3 (x_2-7)^2$.
b. $y = 12 + 3 x_1^{-1} + 2 x_2^{-1}$.
c. $y = 25 + 4 x_1^{1/2} + 6 x_2^{1/2}$.
d. $y = 16 + 6 x_1 + 7 x_2$.
e. $y = 5x_1 + 9x_2 + 4 (x_1 x_2)^{1/2}$.
f. $y = 14 x_1^3 x_2^2$.

(7) Which of the following functions has constant partial *elasticities* (ε_1 and ε_2) ?

a. $y = 8 (x_1-4)^3 (x_2-7)^2$.
b. $y = 12 + 3 x_1^{-1} + 2 x_2^{-1}$.
c. $y = 25 + 4 x_1^{1/2} + 6 x_2^{1/2}$.
d. $y = 16 + 6 x_1 + 7 x_2$.
e. $y = 5x_1 + 9x_2 + 4 (x_1 x_2)^{1/2}$.
f. $y = 14 x_1^3 x_2^2$.

The next three questions refer to the following graph of a level curve, or contour, of the function $y = f(x_1,x_2)$.
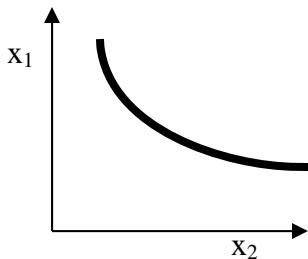
(8) By definition, all points along the curve in this graph have identical values of

a. x_1 .
b. x_2 .
c. y .
d. the marginal rate of substitution.
e. all of the above.
f. none of the above.

(9) According to this graph, if x_1 increases and y is to be held constant, then x_2 must

a. increase.
b. decrease.
c. remain constant.
d. be set equal to zero.
e. cannot be determined from the information given.

(10) Along this level curve, as we move down and to the right, the marginal rate of substitution of x_2 for x_1 (that is, the |slope| of the level curve with x_1 on the vertical axis and x_2 on the horizontal axis) is

a. increasing.
b. diminishing.
c. infinite.
d. constant and equal to zero.
e. constant and equal to one.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals 2 at a particular value of x. Moreover, the elasticity of y with respect to x equals 0.3. Further suppose that x increases by 10 *units*. [Hint: Some of this information is extraneous and not needed to answer this question.]

 a. Will y *increase* or *decrease*?

 b. By about how much?

units

(2) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 5$, and $\partial y / \partial x_2 = 2$, and that the partial elasticities are $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.5$. Further suppose that x_1 increases by 5 *percent* and simultaneously x_2 increases by 2 *percent*. [Hint: Some of this information is extraneous and not needed to answer this question.]

 a. Will y *increase* or *decrease*?

 b. By about how much?

percent

(3) [4 pts] Revenue equals price times quantity sold. Suppose price increases by 3 percent and the quantity sold decreases by 5 percent.

 a. Will revenue *increase* or *decrease*?

 b. By about how much?

%

(4) [4 pts] The average product of labor equals total output divided by total labor input. Suppose total output in the automobile sector increases by 5 percent and total labor input increases by 2 percent.

 a. Will the average product *increase* or *decrease*?

 b. By about how much?

%

(5) [8 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 3$, and $\partial y / \partial x_2 = 2$. First, suppose that x_1 increases by 4 units but x_2 does not change.

   a. Will y *increase* or *decrease*?

   b. By about how much?

units

Now suppose that x_1 increases by 4 units but we want y to remain constant. To keep y constant, we much change the value of x_2 .

   c. Must x_2 *increase* or *decrease*?

   d. By about how much?

units

(6) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 4$, and $\partial y / \partial x_2 = 7$. Now consider a graph of the level curve of this function, with x_1 on the vertical axis and x_2 on the horizontal axis.

   a. Does the level curve of the function slope *up* or *down* at that point?

   b. Give the slope of the level curve at this point.

III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = -x^2 + 10x - 15$.

    a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

    b. Compute the value x^* that maximizes this function.

    c. For what range of values of x does this function slope up? For what range of values does it slope down?

    d. Find the maximum value, y^*, of the function itself.

(2) [Partial elasticities: 6 pts] Suppose $y = x_1^2 (x_2 - 3)^4$.

    a. Find an expression for ε_1 , the partial elasticity of y with respect to x_1. The variable y should *not* appear in your answer.

    b. Find an expression for ε_2 , the partial elasticity of y with respect to x_2. The variable y should *not* appear in your answer.

(3) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = -7 x_1^{-1} - 5 x_2^{-1}$. The arguments x_1 and x_2 are strictly positive.

a. Find an expression for the partial derivative of y with respect to x_1.

b. Find an expression for the partial derivative of y with respect to x_2.

c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

(4) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = (x_1 - 5)^2 (x_2 - 4)^3$. The arguments x_1 and x_2 are strictly positive.

a. Find an expression for the partial derivative of y with respect to x_1 .

b. Find an expression for the partial derivative of y with respect to x_2 .

c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis). Simplify if possible.

IV. CRITICAL THINKING: [4 pts]

Suppose $y = f(x_1, x_2)$. Let ε_1 denote the partial elasticity of y with respect to x_1 and ε_2 denote the partial elasticity of y with respect to x_2 . Suppose $\varepsilon_1 + \varepsilon_2 = 1$. If x_1 and x_2 both simultaneously increase by 3 percent, does y *increase* or *decrease*? By how about much? Justify your answer.

[end of exam]