

EXAMINATION #1 ANSWER KEY “Mathematical Tools”

Version A

I. Multiple choice

(1)b. (2)b. (3)d. (4)e. (5)c. (6)c. (7)a. (8)c. (9)b. (10)f.

II. Short answer

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|-----|-------------|--|
| (1) | a. increase | b. 4 %, using elasticity since change is given in percent. |
| (2) | a. increase | b. 3 units, using derivative since changes are given in units. |
| (3) | a. decrease | b. 1 %. |
| (4) | a. decrease | b. 2 %. |
| (5) | a. down | b. decrease c. 2/5 units, using total derivative. |

III. Problems

- (1) a. $dy/dx = 2x-10$. b. $x^* = 5$.
 c. Slopes up for $x > 5$ because dy/dx is positive.
 Slopes down for $x < 5$ because dy/dx is negative.
 d. $y^* = f(x^*) = f(5) = 9$.
- (2) a. $\epsilon_1 = \frac{4x_1}{x_1+3}$. b. $\epsilon_2 = 2$.
- (3) a. $\epsilon_1 = 1/3$ b. $\epsilon_2 = 1/6$
 c. y will increase by $(1/3) 3\% + (1/6) 3\% = 1.5\%$.
- (4) a. $\frac{\partial y}{\partial x_1} = 2(x_1 - 3)(x_2 - 5)^3$. b. $\frac{\partial y}{\partial x_2} = (x_1 - 3)^2 3(x_2 - 5)^2$.
 c. $MRS = \frac{\partial y / \partial x_2}{\partial y / \partial x_1} = \frac{3(x_1 - 3)}{2(x_2 - 5)}$.
- (5) a. $MRS = \frac{\partial y / \partial x_2}{\partial y / \partial x_1} = \frac{3}{5} \left(\frac{x_1}{x_2} \right)^{1/2}$.
 b. There are many possible answers. Here are a few.
 $y = 5x_1^{1/2} + 3x_2^{1/2} + constant$.
 $y = (5x_1^{1/2} + 3x_2^{1/2}) \times constant$.
 $y = (5x_1^{1/2} + 3x_2^{1/2})^{constant}$.
 $y = \ln(5x_1^{1/2} + 3x_2^{1/2})$.
 $y = \exp(5x_1^{1/2} + 3x_2^{1/2})$.

IV. Critical thinking

$MRS = zero/nonzero = 0$. So level curves must be horizontal lines.

Version B

I. Multiple choice

- (1)d. (2)c. (3)f. (4)b. (5)d. (6)d. (7)c. (8)a. (9)a. (10)b.

II. Short answer

- (1) a. increase b. 12 units, using derivative since change is given in units.
(2) a. increase b. 5 %, using elasticity since change is given in percent.
(3) a. increase b. 2 %.
(4) a. increase b. 4 %.
(5) a. down b. increase c. 5/2 units, using total derivative.

III. Problems

- (1) a. $dy/dx = -2x+6$. b. $x^* = 3$.
c. Slopes up for $x < 3$ because dy/dx is positive.
Slopes down for $x > 3$ because dy/dx is negative.
d. $y^* = f(x^*) = f(3) = 22$.
- (2) a. $\epsilon_1 = 5$. b. $\epsilon_2 = \frac{2x_2}{x_2+4}$.
- (3) a. $\epsilon_1 = 2/3$ b. $\epsilon_2 = 4/3$
c. y will increase by $(2/3) 3\% + (4/3) 3\% = 6\%$.
- (4) a. $\frac{\partial y}{\partial x_1} = (1/2)x_1^{-3/2}$. b. $\frac{\partial y}{\partial x_2} = (1/2)x_2^{-3/2}$.
c. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \left(\frac{x_1}{x_2}\right)^{3/2}$.
- (5) a. $MRS = \frac{\partial y/\partial x_2}{\partial y/\partial x_1} = \frac{3}{5}\left(\frac{x_1}{x_2}\right)^3$.
b. There are many possible answers. Here are a few.
 $y = -5x_1^{-2} - 3x_2^{-2} + constant$.
 $y = (-5x_1^{-2} - 3x_2^{-2}) \times constant$.
 $y = (-5x_1^{-2} - 3x_2^{-2})constant$.
 $y = \exp(-5x_1^{-2} - 3x_2^{-2})$.

IV. Critical thinking

$MRS = \text{nonzero}/\text{zero} = \infty$. So level curves must be vertical lines.

[end of answer key]