

EXAMINATION #1 VERSION B
“Mathematical Tools”
August 27, 2014

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets. As usual in this course, “ $\exp(x)$ ” denotes the exponential function (also written e^x) while “ $\ln(x)$ ” denotes the natural logarithm function (logarithm to base e).

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [3 pts each—30 pts total]

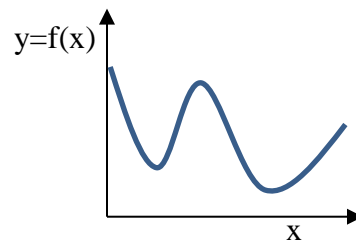
- (1) Suppose the derivative of the function $y = f(x)$ equals zero at a particular value of x . At that point, the graph of the function is
- upward-sloping.
 - downward-sloping.
 - vertical.
 - horizontal.
 - cannot be determined from the information given.

- (2) Suppose $y = 3x^2 + 5x + 7$. Then the derivative of y with respect to x is
- $dy/dx = 6$.
 - $dy/dx = 5$.
 - $dy/dx = 6x + 5$.
 - $dy/dx = 3x + 5$.
 - $dy/dx = 5x + 7$.
 - $dy/dx = 6x^2 + 6x + 7$.

- (3) Which of the following functions has constant slope (or derivative)?
- $y = 7 + (5/x)$.
 - $y = 3 + 5x + (1/2)x^2$.
 - $y = 4x^{-2}$.
 - $y = \ln(2x)$.
 - $y = \exp(3x)$.
 - $y = 3 + 2x$.

- (4) If x increases by 4 percent, then $\ln(x)$ increases by about
- 0.04 percent.
 - 0.04 units.
 - 4 percent.
 - 4 units.
 - $\ln(4)$, or about 1.4 units.

The next question refers to the following graph of $y = f(x)$.



- (5) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at
- no point on the graph.
 - one point on the graph.
 - two points on the graph.
 - three points on the graph.
 - four points on the graph.
 - more than four points on the graph.

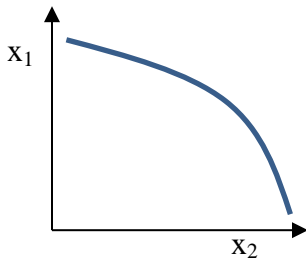
- (6) If $y = 3x$, then the elasticity of y with respect to x equals
- minus one.
 - zero.
 - one-third.
 - one.
 - three.

- (7) Consider the following functions. Which has constant elasticity?
- $y = 7 + (5/x)$.
 - $y = 3 + 5x + (1/2)x^2$.
 - $y = 4x^{-2}$.
 - $y = \ln(2x)$.
 - $y = \exp(3x)$.
 - $y = 3 + 2x$.

- (8) By definition, all points along the curve in this graph have identical values of
- y .
 - the marginal rate of substitution.
 - x_1 .
 - x_2 .
 - all of the above.

- (9) According to this graph, if x_1 decreases and y is to be held constant, then x_2 must
- increase.
 - decrease.
 - remain constant.
 - equal zero.
 - cannot be determined from the information given.

The next two questions refer to the following graph of a level curve, or contour, of the function $y = f(x_1, x_2)$.



- (10) Which of the following functions has constant partial *elasticities* (ϵ_1 and ϵ_2)?
- $y = 2x_1 + 3x_2 + 6(x_1x_2)^{1/2}$.
 - $y = 5x_1^2x_2^3$.
 - $y = 5(x_1+2)^2(x_2+3)^3$.
 - $y = 5 + 2x_1^{-1} + 3x_2^{-1}$.
 - $y = 5 + 2x_1^{1/2} + 3x_2^{1/2}$.
 - $y = 5 + 2x_1 + 3x_2$.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

- (1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals 3 at a particular value of x . Moreover, the elasticity of y with respect to x equals 1.5. Further suppose that x increases by 4 *units*.

- Will y *increase* or *decrease*?
- By about how much?

units

(2) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = -3$, and $\partial y / \partial x_2 = 5$, and that the partial elasticities are $\varepsilon_1 = -0.5$ and $\varepsilon_2 = 1.2$. Suppose that x_1 increases by 2 percent and simultaneously x_2 increases by 5 percent.

a. Will y increase or decrease?

%

b. By approximately how much?

(3) [4 pts] Revenue equals price times quantity sold. Suppose price increases by 5 percent and the quantity sold decreases by 3 percent.

a. Will revenue increase or decrease?

%

b. By about how much?

(4) [4 pts] The capital-labor ratio equals the quantity of capital divided by the quantity of labor. Suppose the quantity of capital increases by 5 percent and the quantity of labor increases by 1 percent.

a. Will the capital-labor ratio increase or decrease?

%

b. By about how much?

(5) [6 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 2$, and $\partial y / \partial x_2 = 5$.

a. Does the level curve of the function slope up or down at that point?

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Further suppose x_2 decreases by 1 unit, but suppose we want to keep the value of y constant.

b. Must x_1 increase or decrease?

units

c. By approximately how much?

III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = -x^2 + 6x + 13$.

- a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

- b. Compute the value x^* that maximizes this function.

- c. For what range of values of x does this function slope up? For what range of values does it slope down?

- d. Find the maximum value, y^* , of the function itself.

(2) [Partial elasticities: 8 pts] Suppose $y = x_1^5 (x_2 + 4)^2$.

- a. Find an expression for ε_1 , the partial elasticity of y with respect to x_1 . The variable y should *not* appear in your answer.

- b. Find an expression for ε_2 , the partial elasticity of y with respect to x_2 . The variable y should *not* appear in your answer.

(3) [Partial elasticities: 8 pts] Suppose $y = x_1^{2/3} x_2^{4/3}$.

a. [2 pts] Find the partial elasticity of y with respect to x_1 .

b. [2 pts] Find the partial elasticity of y with respect to x_2 .

c. [4 pts] Suppose x_1 and x_2 both increase by 3%. Will y increase or decrease? By how much?

(4) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = 6 - x_1^{-1/2} - x_2^{-1/2}$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the partial derivative of y with respect to x_1 .

- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis).

(5) [8 pts] Suppose $y = f(x_1, x_2) = -5/x_1^2 - 3/x_2^2$.

- a. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis).

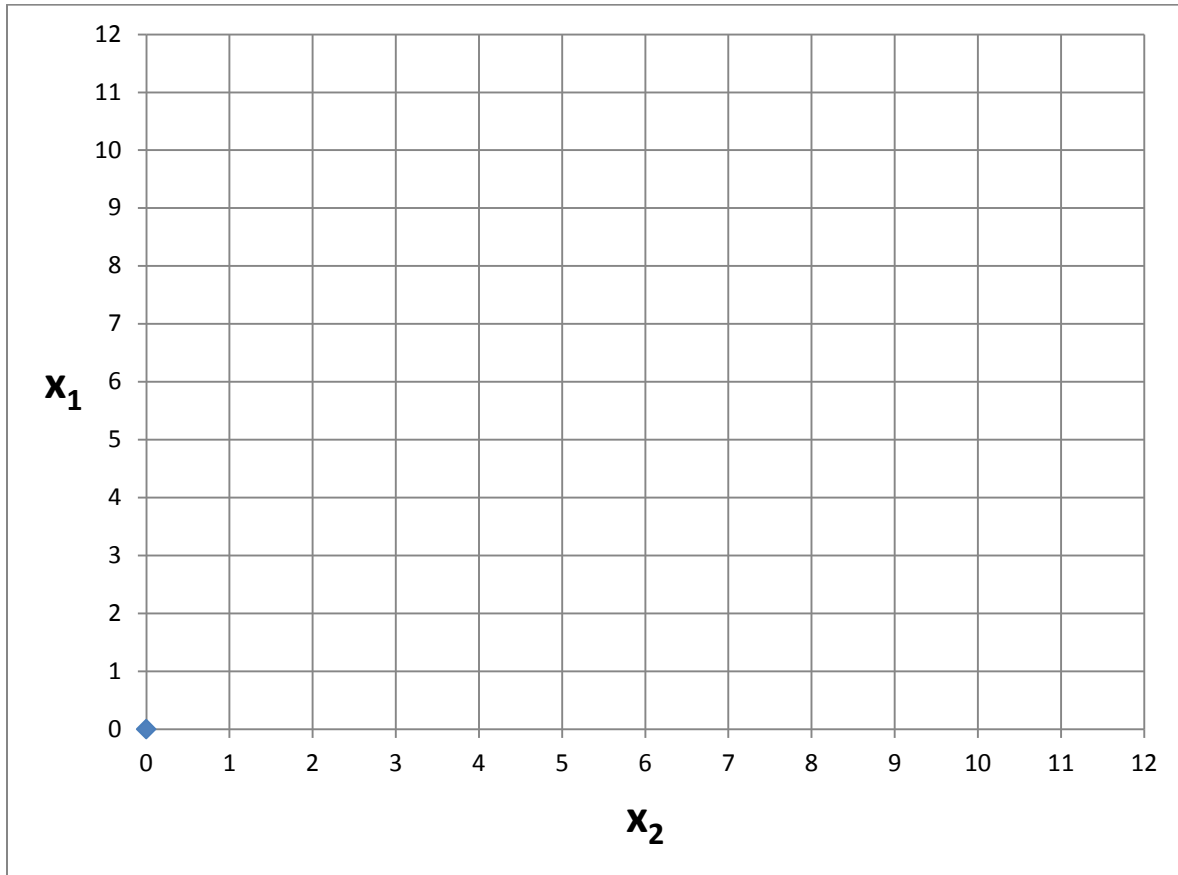


- b. Give an example of a different function for y that has exactly the same formula for the marginal rate of substitution.



IV. CRITICAL THINKING: [4 pts]

(1) Suppose $y = f(x_1, x_2)$. Further suppose $\partial y / \partial x_1$ is always equal to zero, but $\partial y / \partial x_2$ is nonzero. Sketch two level curves of this function in the graph below.



[end of exam]