

EXAMINATION #4 ANSWER KEY

Version A

I. Multiple choice

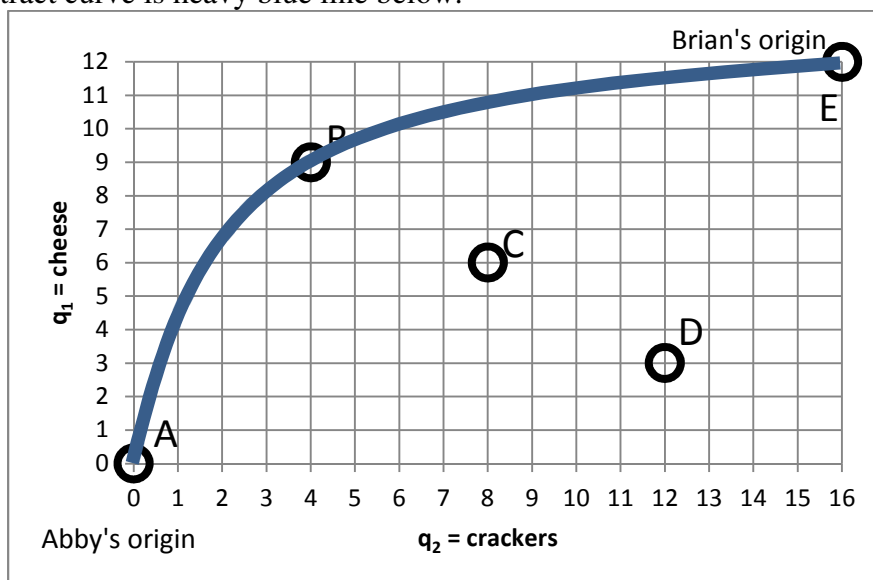
(1)a. (2)d. (3)c. (4)b. (5)d. (6)b. (7)a. (8)b. (9)d.

II. Short answer

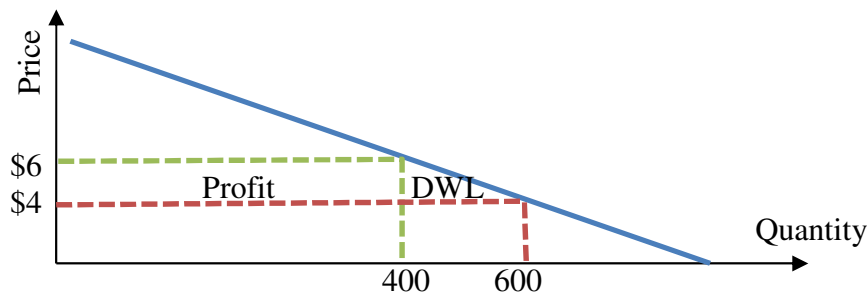
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|-----|---------------------------------------|---|-----------------------------------|
| (1) | a. 3 units of other goods
d. \$18. | b. 1/3 units of food | c. slope = -3 |
| (2) | a. \$1.95 | b. increase | c. \$0.95. |
| (3) | a. \$9 | b. \$4. | |
| (4) | a. $MR = 8 - Q$
c. \$6
f. \$5 | b. MR is straight line with P-intercept = \$8, slope = -1
d. 4 million
g. 6 million | e. \$2 million
h. \$0 million. |

III. Problems

- (1) Note that Abby's $MRS_A = q_{1A} / (3 q_{2A})$ and Brian's $MRS_B = 3 q_{1B} / q_{2B}$.
- Yes** Pareto-efficient, because Abby has nothing, so she cannot be made better off without taking some of Brian's cheese or crackers, which would make Brian worse off.
 - Yes** Pareto-efficient, because $MRS_A = 3/4 = MRS_B$.
 - No**, not Pareto-efficient, because $MRS_A = 1/3 \neq MRS_B = 3$.
 - No**, not Pareto-efficient, because $MRS_A = 1/12 \neq MRS_B = 27/4$.
 - Yes** Pareto-efficient, because Brian has nothing, so he cannot be made better off without taking some of Abby's cheese or crackers, which would make Abby worse off.
 - Contract curve is heavy blue line below.



- (2) a. To minimize total cost, marginal costs must be equal. Now $MC_A = dTC_A/dq_A = 5 + (q_A/20)$, and $MC_B = dTC_B/dq_B = 4 + (q_B/10)$. So set $5 + (q_A/20) = 4 + (q_B/10)$. Then substitute $q_B = 100 - q_A$, to get $5 + (q_A/20) = 4 + ((100 - q_A)/10)$. Solve to get **$q_A = 60$ and $q_B = 40$** .
 b. At these quantities, $MC_A = MC_B = \$8$. So if the market price were **$\$8$** , and each firm took this price as given, then Firm A would maximize its profit by producing $q_A = 60$ and Firm B would maximize its profit by producing $q_B = 40$.
- (3) a. $Rev = P \times Q = 50Q - (Q^2/100)$, so $MR = dRev/dQ = 50 - Q/50$. Set $MR = MC = 10$ and solve to get **$Q = 2000$** . Insert into demand equation to get $P = 50 - (2000/100) = \mathbf{\$30}$.
 b. $Rev = P \times Q = \$30 \times 2000 = \$60,000$. Production cost = $\$10 \times 2000 = \$20,000$. Difference = maximum amount of up-front costs the company would pay for developing and patenting the chip = **$\$40,000$** .
 c. Under competition, $P = MC = \mathbf{\$10}$. Substituting into demand equation $\$10 = 50 - Q/100$, so $Q = \mathbf{4000}$.
 d. $DWL = \text{area of triangle} = \frac{1}{2} \times \$20 \times 2000 = \mathbf{\$20,000}$.
- (4) a. $Rev_1 = P q_1 = \mathbf{10q_1 - (q_1^2/100) - (q_1q_2/100)}$.
 b. $MR_1 = \partial Rev_1(q_1, q_2) / \partial q_1 = \mathbf{10 - 2q_1/100 - q_2/100}$.
 c. Set $MR_1 = MC = \$4$ and solve to get **$q_1^* = 300 - q_2/2$** .
 d. Since $q_1^* = q_2^*$, $q_1^* = 300 - q_1^*/2$. Solving yields **$q_1^* = 200 = q_2^*$** .
 e. $Q = q_1^* + q_2^* = \mathbf{400}$. Substituting into demand equation: $P^* = 10 - (400/100) = \mathbf{\$6}$.
 f. Total revenue = $P^* \times Q^* = \$2400$. Total cost = $AC \times Q^* = \$1600$. Total profit = total revenue - total cost = **$\$800$** .
 g. The efficient level of output lies where marginal cost intersects demand. Find this quantity by setting $MC = \$4 = P = 10 - (Q/100)$ and solving to get $Q = 600$. Deadweight loss is the area between demand and marginal cost, from the Cournot equilibrium quantity $Q^*=400$ to the efficient quantity = 600. This is the area of a triangle, equal to **$\$200$** .



- (5) This game resembles a “Battle of the Sexes” game in that there are two Pareto-optimal outcomes which are also Nash equilibria.
- a. There are two Pareto-optimal outcomes of this game:
1. Retailer A locates uptown, Retailer B locates downtown.
 2. Retailer A locates downtown, Retailer B locates uptown.
- b. Neither player has a dominant strategy, so there are **no** dominant-strategy equilibria.
- c. There are two Nash equilibria:
1. Retailer A locates uptown, Retailer B locates downtown.
 2. Retailer A locates downtown, Retailer B locates uptown.

IV. Critical thinking

(1) This market is not a Cournot duopoly, because firms choose *prices* rather than quantities, but it may be solved using a similar approach.

a. Since Firm A maximizes its own revenue, taking as given the price of the other firm, we should begin by finding an expression for Firm A's revenue as a function of its own price P_A and the other firm's price P_B :

$$\text{Rev}_A = Q_A \times P_A = (120 - 20 P_A + 10 P_B) \times P_A = 120 P_A - 20 P_A^2 + 10 P_A P_B .$$

Maximize this function with respect to P_A by setting its partial derivative equal to zero:

$$0 = \partial \text{Rev}_A / \partial P_A = 120 - 40 P_A + 10 P_B .$$

Solve for P_A in terms of P_B to get Firm A's best reply function:

$$P_A = 3 + (1/4) P_B .$$

b. Following the same procedure for Firm B gives its best reply function:

$$P_B = 3 + (1/4) P_A .$$

Solving these two best reply functions jointly gives $P_A = \$4$ and $P_B = \$4$.

Version B

I. Multiple choice

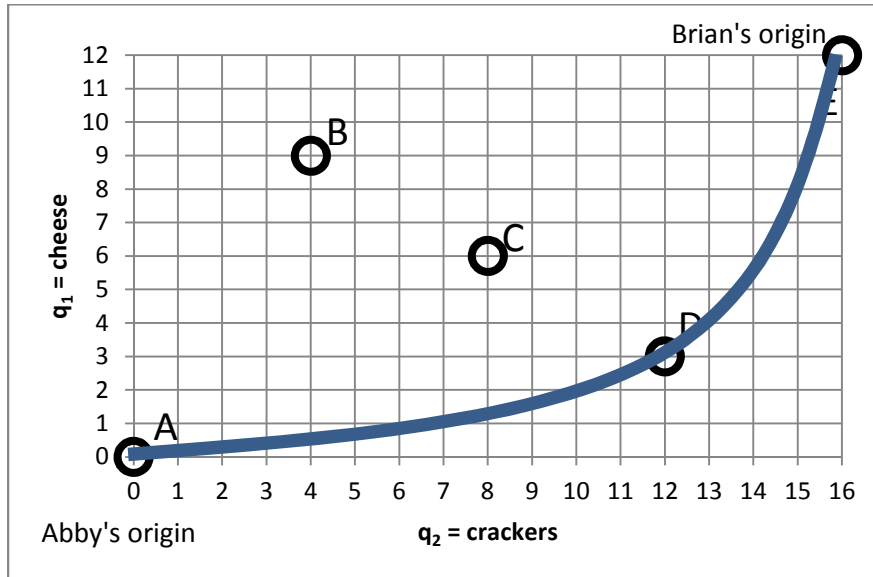
(1)b. (2)a. (3)e. (4)d. (5)b. (6)c. (7)c. (8)a. (9)e.

II. Short answer

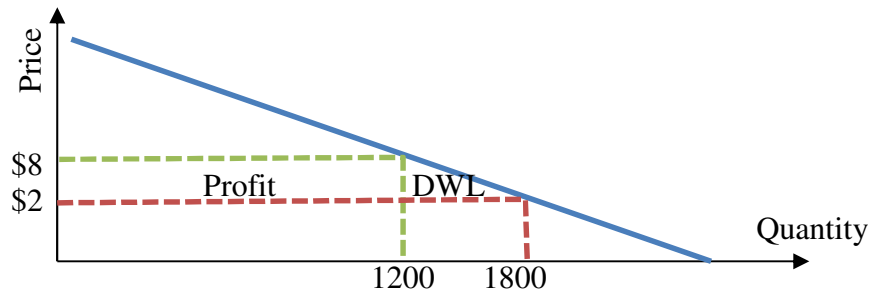
- (1) a. 2 units of other goods b. 1/2 units of food c. slope = -2
d. \$12.
- (2) a. \$0.95 b. decrease c. \$0.05.
- (3) a. \$8 b. \$5.
- (4) a. $MR = 9 - 2Q$ b. MR is straight line with P-intercept = \$9, slope = -2
c. \$6 d. 3 million e. \$3 million
f. \$4 g. 5 million h. \$0 million.

III. Problems

- (1) Note that Abby's $MRS_A = 3 q_{1A} / q_{2A}$ and Brian's $MRS_B = q_{1B} / (3 q_{2B})$.
- a. **Yes** Pareto-efficient, because Abby has nothing, so she cannot be made better off without taking some of Brian's cheese or crackers, which would make Brian worse off.
- b. **No**, not Pareto-efficient, because $MRS_A = 27/4 \neq MRS_B = 1/12$.
- c. **No**, not Pareto-efficient, because $MRS_A = 3 \neq MRS_B = 1/3$.
- d. **Yes** Pareto-efficient, because $MRS_A = 3/4 = MRS_B$.
- e. **Yes** Pareto-efficient, because Brian has nothing, so he cannot be made better off without taking some of Abby's cheese or crackers, which would make Abby worse off.
- f. Contract curve is heavy blue line below.



- (2) a. To minimize total cost, marginal costs must be equal. Now $MC_A = dTC_A/dq_A = 12 + (q_A/20)$, and $MC_B = dTC_B/q_B = 13 + (q_B/40)$. So set $12 + (q_A/20) = 13 + (q_B/40)$. Then substitute $q_B = 200 - q_A$, to get $12 + (q_A/20) = 13 + ((200 - q_A)/40)$. Solve to get **$q_A = 80$ and $q_B = 120$** .
- b. At these quantities, $MC_A = MC_B = \$16$. So if the market price were **$\$16$** , and each firm took this price as given, then Firm A would maximize its profit by producing $q_A = 80$ and Firm B would maximize its profit by producing $q_B = 120$.
- (3) a. $Rev = P \times Q = 25Q - (Q^2/50)$, so $MR = dRev/dQ = 25 - Q/25$. Set $MR = MC = 5$ and solve to get **$Q = 500$** . Insert into demand equation to get $P = 25 - (500/50) = \mathbf{\$15}$.
- b. $Rev = P \times Q = \$15 \times 500 = \7500 . Production cost = $\$5 \times 500 = \2500 . Difference = maximum amount of up-front costs the company would pay for developing and patenting the chip = **$\$5000$** .
- c. Under competition, $P = MC = \mathbf{\$5}$. Substituting into demand equation $\$10 = 25 - Q/50$, so $Q = \mathbf{1000}$.
- d. $DWL = \text{area of triangle} = \frac{1}{2} \times \$20 \times 2000 = \mathbf{\$2500}$.
- (4) a. $Rev_1 = P q_1 = \mathbf{20q_1 - (q_1^2/100) - (q_1q_2/100)}$.
- b. $MR_1 = \partial Rev_1(q_1, q_2) / \partial q_1 = \mathbf{20 - 2q_1/100 - q_2/100}$.
- c. Set $MR_1 = MC = \$2$ and solve to get $q_1^* = \mathbf{900 - q_2/2}$.
- d. Since $q_1^* = q_2^*$, $q_1^* = 900 - q_1^*/2$. Solving yields $q_1^* = \mathbf{600} = q_2^*$.
- e. $Q = q_1^* + q_2^* = \mathbf{1200}$. Substituting into demand equation: $P^* = 20 - (1200/100) = \mathbf{\$8}$.
- f. Total revenue = $P^* \times Q^* = \$9600$. Total cost = $AC \times Q^* = \$2400$. Total profit = total revenue - total cost = **$\$7200$** .
- g. The efficient level of output lies where marginal cost intersects demand. Find this quantity by setting $MC = \$2 = P = 20 - (Q/100)$ and solving to get $Q = 1800$. Deadweight loss is the area between demand and marginal cost, from the Cournot equilibrium quantity $Q^*=1200$ to the efficient quantity = 1800. This is the area of a triangle, equal to **$\$1800$** .



- (5) This game resembles a “Prisoners’ Dilemma” game in that there are three Pareto-optimal outcomes, but the dominant-strategy equilibrium is not one of them.
- a. There are three Pareto-optimal outcomes of this game:
 1. Firm A advertises, Firm B does not advertise.
 2. Firm A does not advertise, Firm B advertises.
 3. Neither firm advertises
 - b. Each firm’s dominant strategy is to advertise, so the dominant-strategy equilibrium is
 1. Both firms advertise.
 - c. The only Nash equilibrium is the dominant-strategy equilibrium:
 1. Both firms advertise.

IV. Critical thinking

Same as Version A.

[end of answer key]