ECON 173 - Intermediate Microeconomic Analysis
Drake University, Fall 2013
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Signature: _____

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EXAMINATION #1 VERSION B
"Mathematical Tools"
September 10, 2013

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets. As usual in this course, "*exp(x)*" denotes the exponential function (also written e^x) while "*ln(x)*" denotes the natural logarithm function (logarithm to base e).

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [3 pts each—30 pts total]

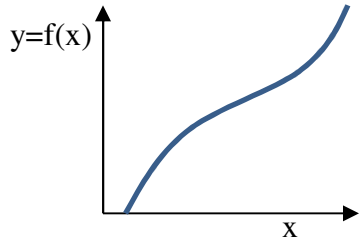
(1) Suppose $y = 7x^2 + 4x + 3$. Then the derivative of y with respect to x is
a. $dy/dx = 7x + 4$.
b. $dy/dx = 4$.
c. $dy/dx = 14x + 4$.
d. $dy/dx = 14x + 7$.
e. $dy/dx = 7x^2 + 4x + 3$.
f. $dy/dx = 3$.

(2) Suppose $y = 3(3x+9)^2$. Then the derivative of y with respect to x is
a. $dy/dx = 3(3x+9)$.
b. $dy/dx = 6(3x+9)^2$.
c. $dy/dx = 2(3x+9)$.
d. $dy/dx = 18(3x+9)$.
e. $dy/dx = (9x+27)^2$.
f. $dy/dx = 6(3x+9)^2 3x$.

(3) Which of the following functions has constant slope (or derivative)?
a. $y = 5x^{-2}$.
b. $y = 5 + 3x$.
c. $y = 5 + (3/x)$.
d. $y = 5 + 3x + 3x^2$.
e. $y = 5\ln(3x)$.
f. All of the above.

(4) Suppose we wish to maximize the function $y = f(x)$, which is continuously differentiable. Assuming there are no restrictions on the possible values of x, the maximizing value x^* must satisfy
a. $dy/dx = 0$, if $x = x^*$.
b. $x^* = 0$.
c. $f(x^*) = 0$.
d. $d^2y/dx^2 = 0$, if $x = x^*$.
e. All of the above.

The next question refers to the following graph of $y = f(x)$.

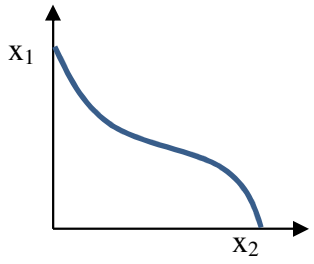


(5) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at
a. no point on the graph.
b. one point on the graph.
c. two points on the graph.
d. three points on the graph.
e. four points on the graph.
f. more than four points on the graph.

(6) If y is proportional to x, then the elasticity of y with respect to x equals
a. negative infinity.
b. x.
c. negative one.
d. zero.
e. one-half.
f. one.
g. cannot be determined from information given.

(7) Which of the following functions has constant elasticity?
a. $y = 5\,x^{-2}$.
b. $y = 5 + 3\,x$.
c. $y = 5 + (3/x)$.
d. $y = 5 + 3\,x + 3\,x^2$.
e. $y = 5\,\ln(3x)$.
f. All of the above.

The next two questions refer to the following graph of a level curve, or contour, of the function $y = f(x_1,x_2)$.



(8) As we move along this curve down and to the right, what remains unchanged?
a. x_1 .
b. x_2 .
c. both x_1, and x_2 .
d. the marginal rate of substitution.
e. y.
f. All of the above remain unchanged.

(9) According to this graph, if x_2 *increases* and y is to be held constant, then x_1 must
a. equal zero.
b. increase.
c. decrease.
d. remain constant.
e. cannot be determined from the information given.

(10) Which of the following functions has constant partial elasticities (ε_1 and ε_2) ?
a. $y = 8 + 3\,x_1 + 4\,x_2$.
b. $y = 8x_1 + 3x_2 + 4\,(x_1x_2)^{1/2}$.
c. $y = 8 + 3\,x_1^{-1} + 4\,x_2^{-1}$.
d. $y = 8 + 3\,x_1^{1/2} + 4\,x_2^{1/2}$.
e. $y = 8\,x_1^{3}\,x_2^{4}$.
f. $y = 8\,(x_1-3)^2\,(x_2-4)^2$.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals –2 at a particular value of x. Moreover, the elasticity of y with respect to x equals –0.6. Suppose that x increases by 5 *percent*.

a. Will y *increase* or *decrease*?

c. By approximately how much?

percent

(2) [4 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = -3$, and $\partial y/\partial x_2 = 5$, and that the partial elasticities are $\varepsilon_1 = -0.8$ and $\varepsilon_2 = 1.2$. Suppose that x_1 increases by 10 *units* and simultaneously x_2 increases by 2 *units*.

a. Will y *increase* or *decrease*?

b. By approximately how much? units

(3) [6 pts] Consider the function $y = f(x_1,x_2)$. Suppose at a particular point, $\partial y/\partial x_1 = 2$, and $\partial y/\partial x_2 = 3$.

a. Does the level curve of the function slope *up* or *down* at that point?

Further suppose x_2 increases by 1 unit, but suppose we want to keep the value of y constant.

b. Must x_1 *increase* or *decrease*?

c. By approximately how much? units

(4) [4 pts] The so-called "wage bill" equals the weekly wage times the number of workers employed. Suppose the wage decreases by 3 percent and the number of workers increases by 1 percent.

a. Will the wage bill *increase* or *decrease*?

b. By approximately how much? %

(5) [4 pts] Average cost equals total cost divided by total output. Suppose total output increases by 5 percent and total cost increases by 2 percent.

a. Will the average cost *increase* or *decrease*?

b. By approximately how much? %

III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = 2x^2 - 100x + 365$.

a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

b. Compute the value x* that minimizes this function.

(2) [Partial elasticities: 8 pts] Suppose $y = (x_1 + 3)^4 x_2^3$.

a. Find an expression for ε_1 , the partial elasticity of y with respect to x_1. The variable y should *not* appear in your answer.

b. Find an expression for ε_2 , the partial elasticity of y with respect to x_2. The variable y should *not* appear in your answer.

(3) [MRS: 12 pts] Suppose $y = f(x_1,x_2) = -4\ x_1^{-1/2} - 6\ x_2^{-1/2}$. The arguments x_1 and x_2 are strictly positive.

a. Find an expression for the partial derivative of y with respect to x_1.

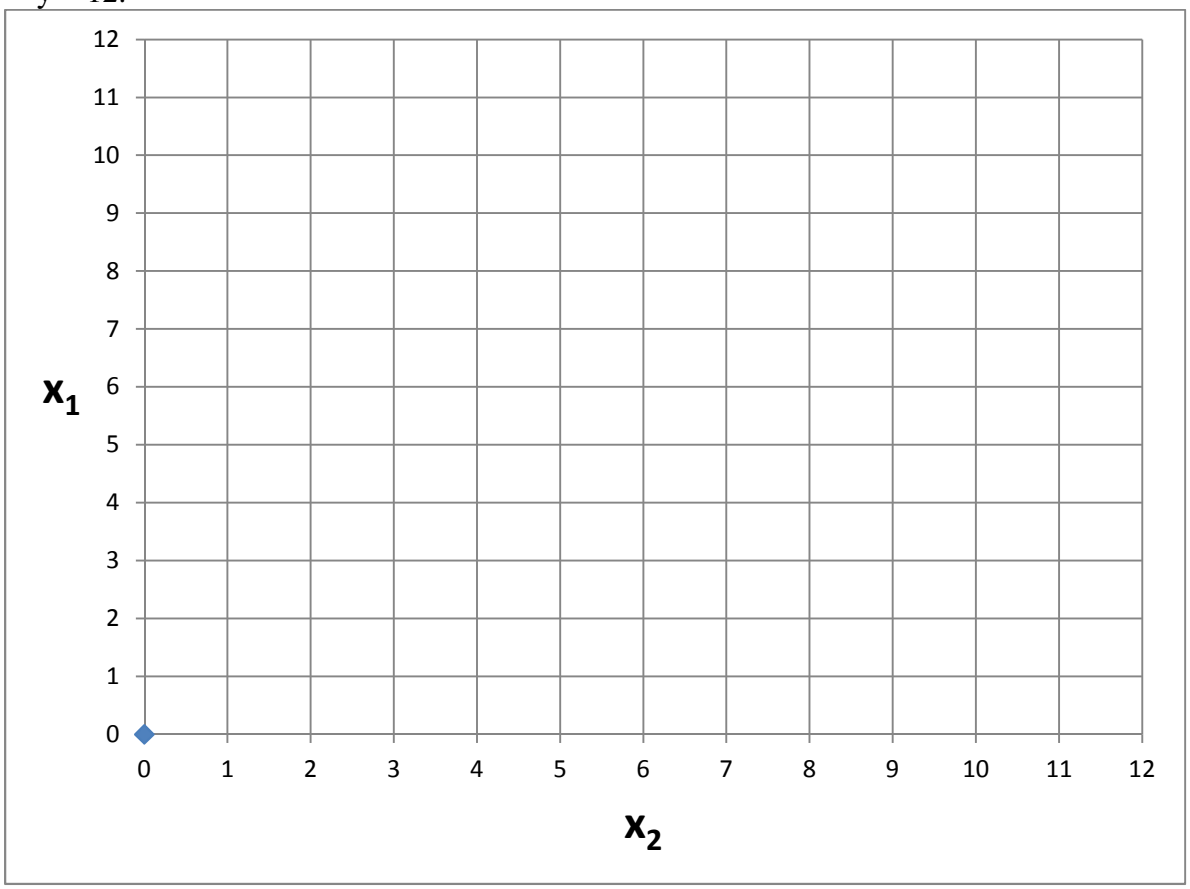
b. Find an expression for the partial derivative of y with respect to x_2.

c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis).

(4) [MRS: 8 pts] Consider the function $y = x_1 + 4\,x_2$.

a. Give the marginal rate of substitution of x_2 for x_1 (that is, is the |slope| of the level curve with x_1 on the vertical axis and x_2 on the horizontal axis) for this function.

b. Carefully draw the level curve of this function when $y = 8$ and the level curve when $y = 12$.



(5) [MRS: 8 pts] Suppose $y = f(x_1,x_2) = x_1^4 (x_2 + 3)^3$. The arguments x_1 and x_2 are strictly positive.

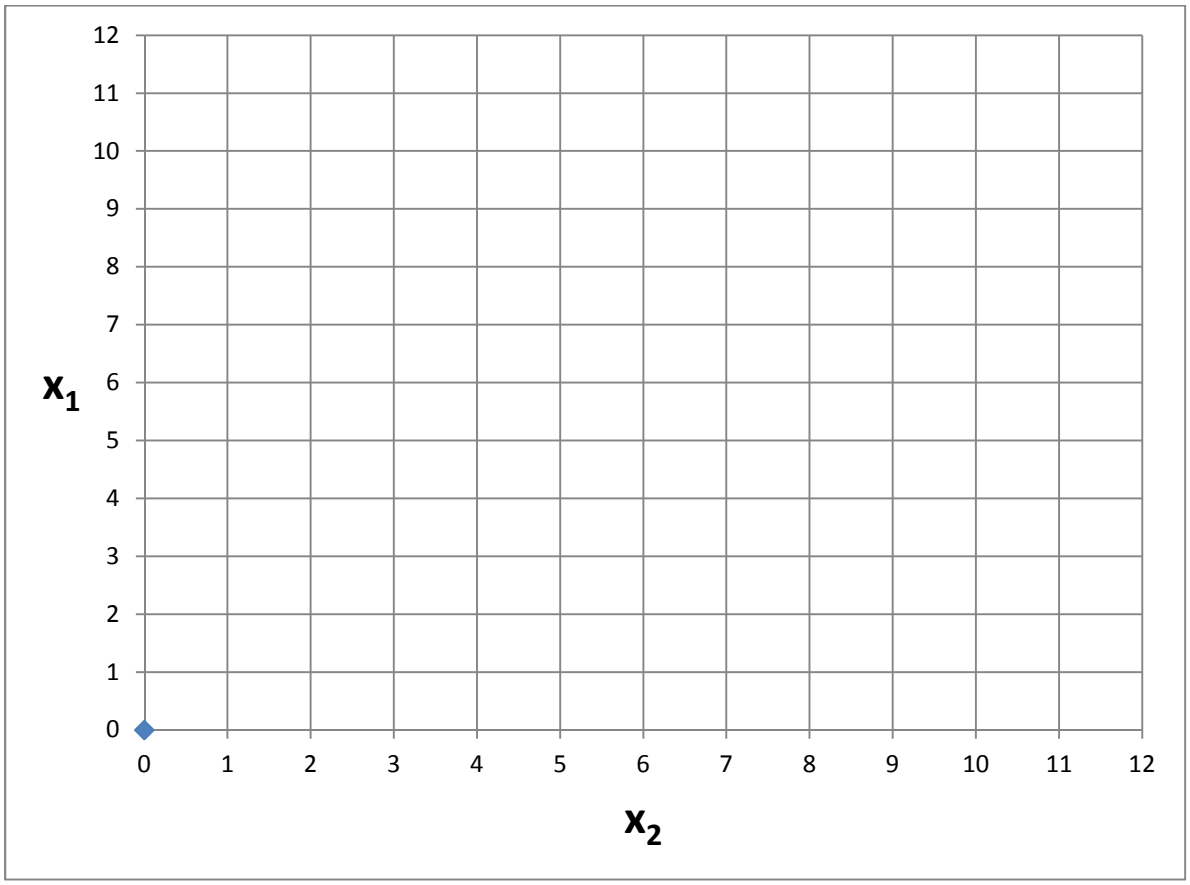
a. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y, with x_1 on the vertical axis and x_2 on the horizontal axis).

b. Give an example of a different function $y = g(x_1,x_2)$ that has exactly the same formula for the marginal rate of substitution as $f(x_1,x_2)$.

$g(x_1,x_2)$ =

IV. CRITICAL THINKING: [4 pts]

(1) Suppose $y = f(x_1,x_2)$. Further suppose $\partial y/\partial x_2$ is always twice as large as $\partial y/\partial x_1$. Sketch two level curves of this function in the graph below.



[end of exam]