

EXAMINATION #1 VERSION B
“Mathematical Tools”
September 12, 2012

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets. As usual in this course, “ $exp(x)$ ” denotes the exponential function (also written e^x) while “ $ln(x)$ ” denotes the natural logarithm function (logarithm to base e).

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [3 pts each—30 pts total]

(1) Suppose $y = 2x^2 + 3x + 11$. Then the derivative of y with respect to x is

- a. $dy/dx = 4$.
- b. $dy/dx = 3$.
- c. $dy/dx = 2x + 3$.
- d. $dy/dx = 3x + 11$.
- e. $dy/dx = 4x + 3$.
- f. $dy/dx = 4x^2 + 3x + 11$.

(2) Suppose $y = 2(3x-5)^4$. Then the derivative of y with respect to x is

- a. $dy/dx = 4$.
- b. $dy/dx = 8$.
- c. $dy/dx = 24(3x-5)^3$.
- d. $dy/dx = 8(3x-5)^3$.
- e. $dy/dx = 6(3x-5)^4$.
- f. $dy/dx = 2(3x-5)^3$.

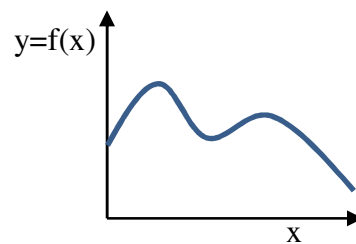
(3) Which of the following functions has constant slope (or derivative)?

- a. $y = 5 \ln(4x)$.
- b. $y = 5 + 4x$.
- c. $y = 5x^{-4}$.
- d. $y = 5 + (4/x)$.
- e. $y = 5 + 4x + 2x^2$.
- f. All of the above.

(4) Suppose we wish to maximize the function $y = f(x)$, which is continuously differentiable. Assuming there are no restrictions on the possible values of x , the maximizing value x^* must satisfy

- a. $d^2y/dx^2 = 0$, if $x = x^*$.
- b. $dy/dx = 0$, if $x = x^*$.
- c. $f(x^*) = 0$.
- d. $x^* = 0$.
- e. All of the above.

The next question refers to the following graph of $y = f(x)$.



(5) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at

- a. no point on the graph.
- b. one point on the graph.
- c. two points on the graph.
- d. three points on the graph.
- e. four points on the graph.
- f. more than four points on the graph.

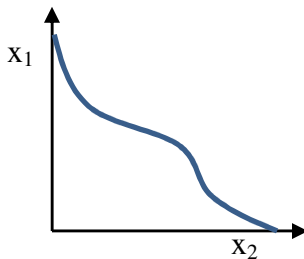
- (6) If y is proportional to x , then the elasticity of y with respect to x equals
- x .
 - zero.
 - one-half.
 - one.
 - two.
 - cannot be determined from information given.

- (7) Which of the following functions has constant elasticity?
- $y = 5 \ln(4x)$.
 - $y = 5 + 4x$.
 - $y = 5x^{-4}$.
 - $y = 5 + (4/x)$.
 - $y = 5 + 4x + 2x^2$.
 - All of the above.

- (8) As we move along this curve down and to the right, what remains unchanged?
- x_1 .
 - x_2 .
 - both x_1 , and x_2 .
 - y .
 - the marginal rate of substitution.
 - All of the above remain unchanged.

- (9) According to this graph, if x_1 decreases and y is to be held constant, then x_2 must
- equal zero.
 - increase.
 - decrease.
 - remain constant.
 - cannot be determined from the information given.

The next two questions refer to the following graph of a level curve, or contour, of the function $y = f(x_1, x_2)$.



- (10) Which of the following functions has constant partial elasticities (ϵ_1 and ϵ_2)?
- $y = 17 + 2x_1^{1/2} + 3x_2^{1/2}$.
 - $y = 17x_1^2x_2^3$.
 - $y = 17(x_1-2)^2(x_2-3)^2$.
 - $y = 17 + 2x_1 + 3x_2$.
 - $y = 17x_1 + 2x_2 + 3(x_1x_2)^{1/2}$.
 - $y = 17 + 2x_1^{-1} + 3x_2^{-1}$.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [4 pts] Suppose the derivative of the function $y = f(x)$ equals 8 at a particular value of x . Moreover, the elasticity of y with respect to x equals 0.4. Suppose that x increases by 1.5 units.

- Will y increase or decrease?
- By approximately how much?

units

(2) [4 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 4$, and $\partial y / \partial x_2 = -3$, and that the partial elasticities are $\epsilon_1 = 0.3$ and $\epsilon_2 = -1.5$. Suppose that x_1 increases by 4 percent and x_2 increases by 6 percent.

c. Will y *increase* or *decrease*?

%

d. By approximately how much?

(3) [6 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 3$, and $\partial y / \partial x_2 = 6$.

a. Does the level curve of the function slope *up* or *down* at that point?

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Further suppose x_2 increases by 1 unit, but suppose we want to keep the value of y constant.

b. Must x_1 *increase* or *decrease*?

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c. By approximately how much?

units

(4) [4 pts] Revenue equals price times quantity sold. Suppose price increases by 4 percent and the quantity sold decreases by 1 percent.

a. Will revenue *increase* or *decrease*?

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b. By approximately how much?

%

(5) [4 pts] Average cost equals total cost divided by total output. Suppose total output increases by 5 percent and total cost increases by 2 percent.

a. Will the average cost *increase* or *decrease*?

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b. By approximately how much?

%

III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = -5x^2 + 50x + 1760$.

a. Find an expression (in terms of x) for the derivative of y with respect to x (dy/dx).

b. Compute the value x^* that maximizes this function.

(2) [Partial elasticities: 8 pts] Suppose $y = x_1^3 (x_2 - 5)^2$.

- a. Find an expression for ϵ_1 , the partial elasticity of y with respect to x_1 . The variable y should *not* appear in your answer.

- b. Find an expression for ϵ_2 , the partial elasticity of y with respect to x_2 . The variable y should *not* appear in your answer.

(3) [MRS: 12 pts] Suppose $y = f(x_1, x_2) = 2x_1^{1/2} + 6x_2^{1/2}$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the partial derivative of y with respect to x_1 .

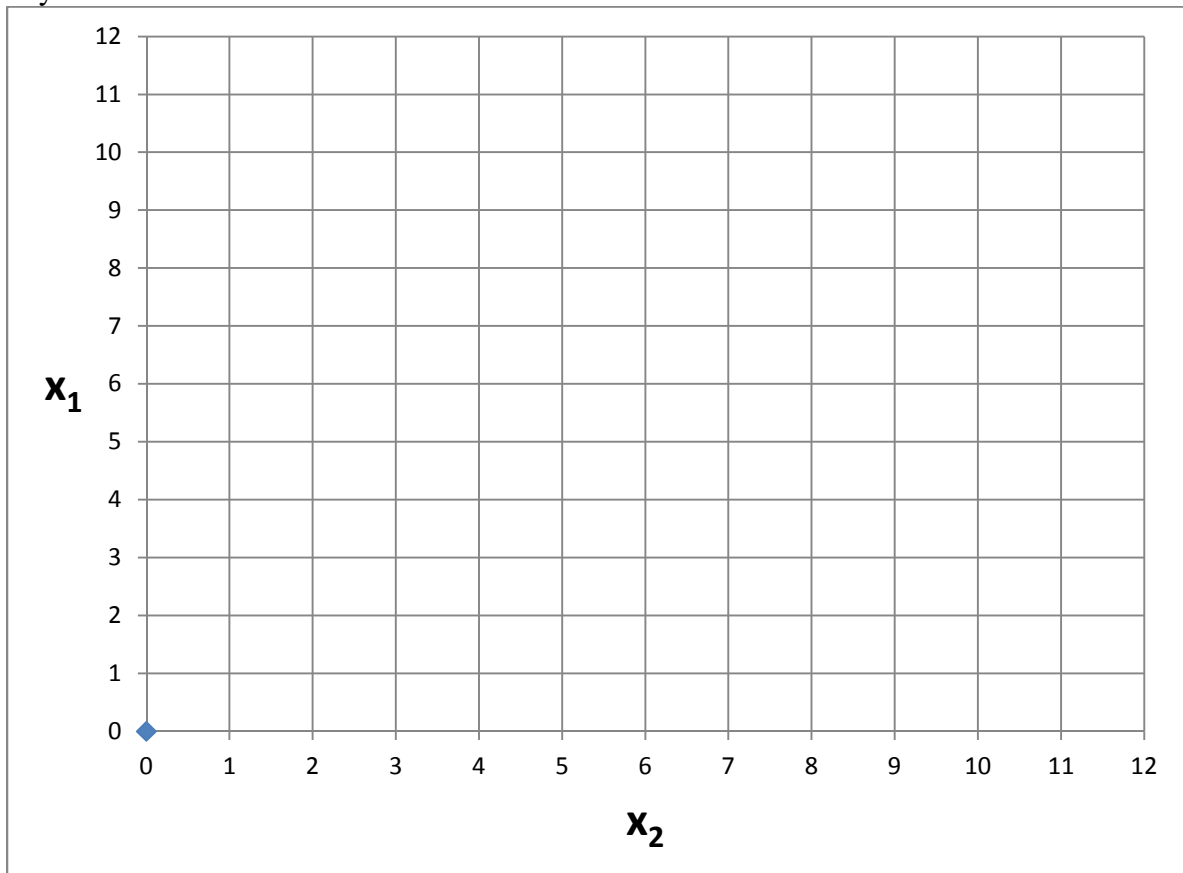
- b. Find an expression for the partial derivative of y with respect to x_2 .

- c. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the slope of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis).

(4) [MRS: 8 pts] Consider the function $y = x_1 + 3 x_2$.

- a. Give the marginal rate of substitution of x_2 for x_1 (that is, is the slope of the level curve with x_1 on the vertical axis and x_2 on the horizontal axis) for this function.

- b. Carefully draw the level curve of this function when $y = 12$ and the level curve when $y = 6$.



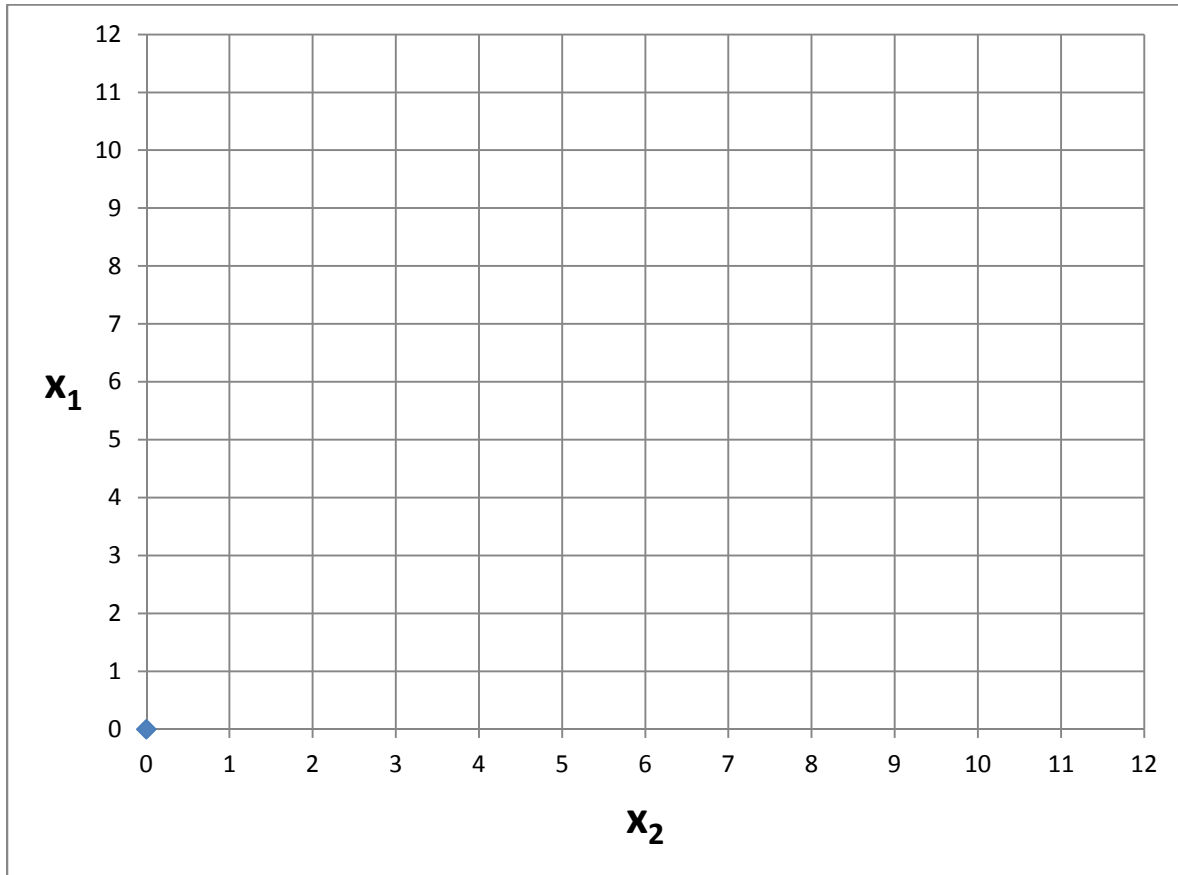
(5) [MRS: 8 pts] Suppose $y = f(x_1, x_2) = (x_1 + 4) x_2^3$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the slope of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis).

- b. Give an example of a different function $y = g(x_1, x_2)$ that has exactly the same formula for the marginal rate of substitution as $f(x_1, x_2)$.

IV. CRITICAL THINKING: [4 pts]

(1) Suppose $y = f(x_1, x_2)$. Further suppose $\partial y / \partial x_1$ is always equal to zero, but $\partial y / \partial x_2$ is always positive. Sketch two level curves of this function in the graph below.



[end of exam]