

EXAMINATION #1 VERSION C
“Mathematical Tools”
September 7, 2011

INSTRUCTIONS: This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets. As usual in this course, “ $\exp(x)$ ” denotes the exponential function (also written e^x) while “ $\ln(x)$ ” denotes the natural logarithm function (logarithm to base e).

I. MULTIPLE CHOICE: Circle the one best answer to each question. Use margins for scratch work. [3 pts each—33 pts total]

(1) Suppose $y = 5x^2 + 4x + 3$. Then the derivative of y with respect to x is

- a. $dy/dx = 5x^2 + 4x + 3$.
- b. $dy/dx = 5x + 4$.
- c. $dy/dx = 4x + 3$.
- d. $dy/dx = 10x + 4$.
- e. $dy/dx = 5$.
- f. $dy/dx = 4$.

(2) Suppose $y = 2(3x+9)^{-2}$. Then the derivative of y with respect to x is

- a. $dy/dx = 2$.
- b. $dy/dx = -4$.
- c. $dy/dx = -12(3x+9)^{-3}$.
- d. $dy/dx = 2(3x+9)^{-3}$.
- e. $dy/dx = (3x+9)^{-3}$.
- f. $dy/dx = -4(3x+9)^{-3}$.

(3) Which of the following functions has constant slope (or derivative)?

- a. $y = 2 \ln(5x)$.
- b. $y = 2 \exp(5x)$.
- c. $y = 2 + 5x$.
- d. $y = 2 + (5/x)$.
- e. $y = 2 + 5x + 9x^2$.
- f. $y = 2x^{-5}$.

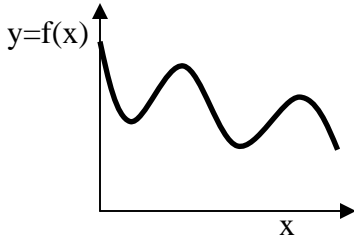
(4) If x increases by 4 percent, then $\ln(x)$ increases by about

- a. 4 units.
- b. 0.04 units.
- c. $\ln(4)$, or about 1.39 units.
- d. 0.04 percent.
- e. 4 percent.

(5) Suppose we wish to maximize the function $y = f(x)$, which is continuously differentiable. Assuming there are no restrictions on the possible values of x , the maximizing value x^* must satisfy

- a. $d^2y/dx^2 = 0$, if $x = x^*$.
- b. $dy/dx = 0$, if $x = x^*$.
- c. $f(x^*) = 0$.
- d. $x^* = 0$.
- e. All of the above.

The next question refers to the following graph of $y = f(x)$.

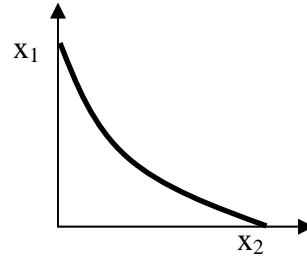


- (6) In this graph, the derivative of y with respect to x (that is, df/dx) equals zero at
- no point on the graph.
 - one point on the graph.
 - two points on the graph.
 - three points on the graph.
 - four points on the graph.
 - more than four points on the graph.

- (7) Suppose y and x are strictly positive variables. If the derivative (dy/dx) is *negative*, then the elasticity of y with respect to x
- must be positive.
 - must be negative.
 - must be zero.
 - can be positive or negative but not zero.
 - can be positive, negative, or zero.

- (8) Which of the following functions has constant elasticity?
- $y = 2 \ln(5x)$.
 - $y = 2 \exp(5x)$.
 - $y = 2 + 5x$.
 - $y = 2 + (5/x)$.
 - $y = 2 + 5x + 9x^2$.
 - $y = 2x^{-5}$.

The next two questions refer to the following graph of a level curve, or contour, of the function $y = f(x_1, x_2)$.



- (9) As we move along this curve down and to the right, which is diminishing?
- y .
 - x_2 .
 - both x_1 , and x_2 .
 - the marginal rate of substitution.
 - all of the above.

- (10) According to this graph, if x_2 increases and y is to be held constant, then x_1 must
- decrease.
 - remain constant.
 - increase.
 - equal zero.
 - cannot be determined from the information given.

- (11) Which of the following functions has constant partial elasticities (ϵ_1 and ϵ_2)?
- $y = 8 + 3x_1^{-1} + 2x_2^{-1}$.
 - $y = 8 + 3x_1^{1/2} + 2x_2^{1/2}$.
 - $y = 8x_1^3x_2^2$.
 - $y = 8(x_1-3)^2(x_2-2)^2$.
 - $y = 8 + 3x_1 + 2x_2$.
 - $y = 8x_1 + 3x_2 + 2(x_1x_2)^{1/2}$.

II. SHORT ANSWER: Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [8 pts] Suppose the derivative of the function $y = f(x)$ equals 2 at a particular value of x . Moreover, the elasticity of y with respect to x equals 0.8.

First, suppose that x increases by 0.5 units.

a. Will y *increase* or *decrease*?

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b. By about how much?

units

Alternatively, suppose that x decreases by 5 percent.

c. Will y *increase* or *decrease*?

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d. By about how much?

%

(2) [8 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 5$, and $\partial y / \partial x_2 = 3$, and that the partial elasticities are $\epsilon_1 = 1.5$ and $\epsilon_2 = 2$.

First suppose that x_1 decreases by 0.4 and simultaneously x_2 increases by 0.5.

a. Will y *increase* or *decrease*?

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b. By about how much?

units

Alternatively, suppose that x_1 increases by 4 percent and x_2 decreases by 2 percent.

c. Will y *increase* or *decrease*?

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d. By about how much?

%

(3) [6 pts] Consider the function $y = f(x_1, x_2)$. Suppose at a particular point, $\partial y / \partial x_1 = 4$, and $\partial y / \partial x_2 = 3$.

a. Does the level curve of the function slope *up* or *down* at that point?

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Further suppose x_2 increases by 1 unit, but suppose we want to keep the value of y constant.

b. Must x_1 *increase* or *decrease*?

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c. By about how much?

units

(4) [4 pts] Revenue equals price times quantity sold. Suppose price increases by 3 percent and the quantity sold decreases by 1 percent.

a. Will revenue *increase* or *decrease*?

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b. By about how much?

%

(5) [4 pts] Average cost equals total cost divided by total output. Suppose total output increases by 2 percent and total cost increases by 5 percent.

a. Will the average cost *increase* or *decrease*?

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b. By about how much?

%

III. PROBLEMS: Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function $y = f(x) = 2x^{3/2} - 15x$.

a. Find a formula (in terms of x) for the derivative of y with respect to x (dy/dx).

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b. Compute the value x^* that maximizes this function.

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(2) [Partial elasticities: 8 pts] Suppose $y = x_1^2 (x_2 + 3)^5$.

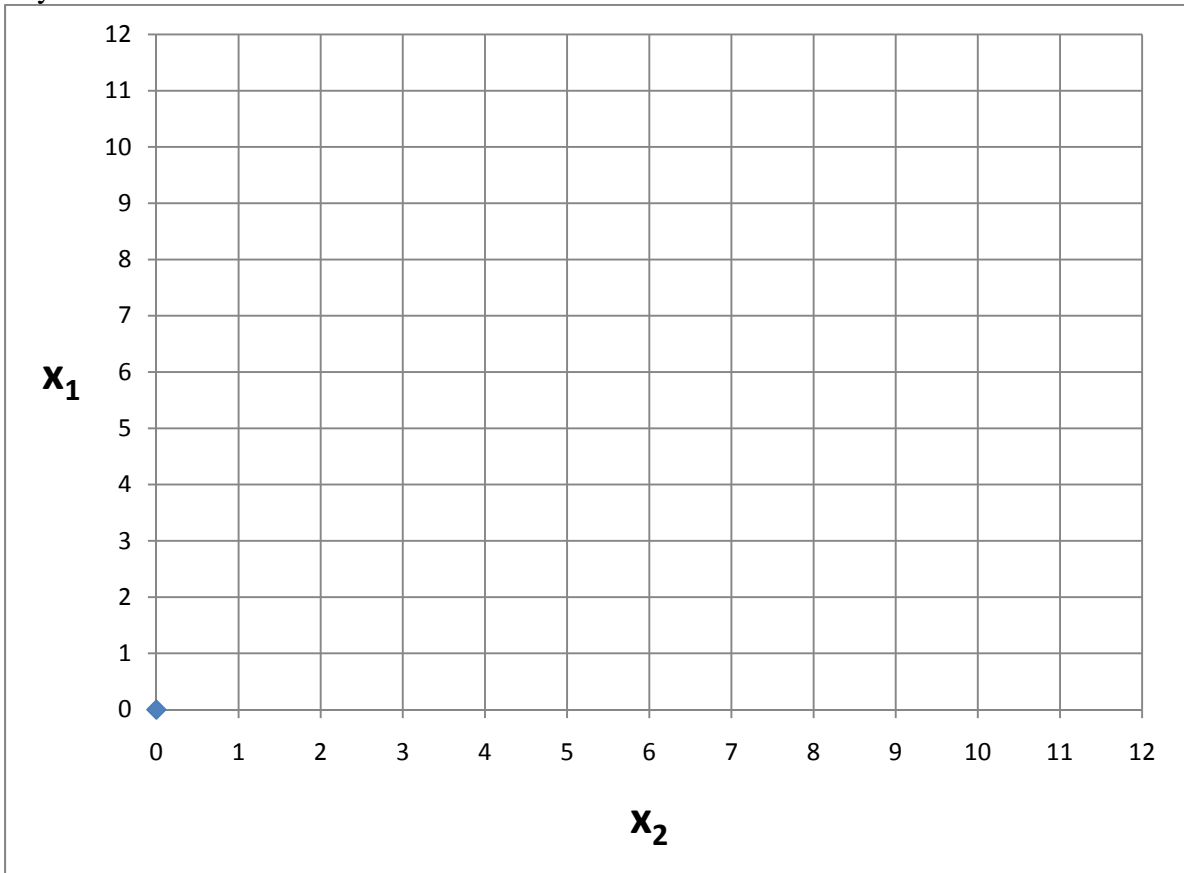
- a. Find a formula for ε_1 , the partial elasticity of y with respect to x_1 . Express your answer in terms of x_1 and x_2 alone, not y .

- b. Find a formula for ε_2 , the partial elasticity of y with respect to x_2 . Express your answer in terms of x_1 and x_2 alone, not y .

(3) [MRS: 8 pts] Consider the function $y = x_1 + 0.5 x_2$.

- a. Give the marginal rate of substitution of x_2 for x_1 (that is, is the |slope| of the level curve with x_1 on the vertical axis and x_2 on the horizontal axis) for this function.

- b. Carefully draw the level curve of this function when $y = 1$ and the level curve when $y = 3$.



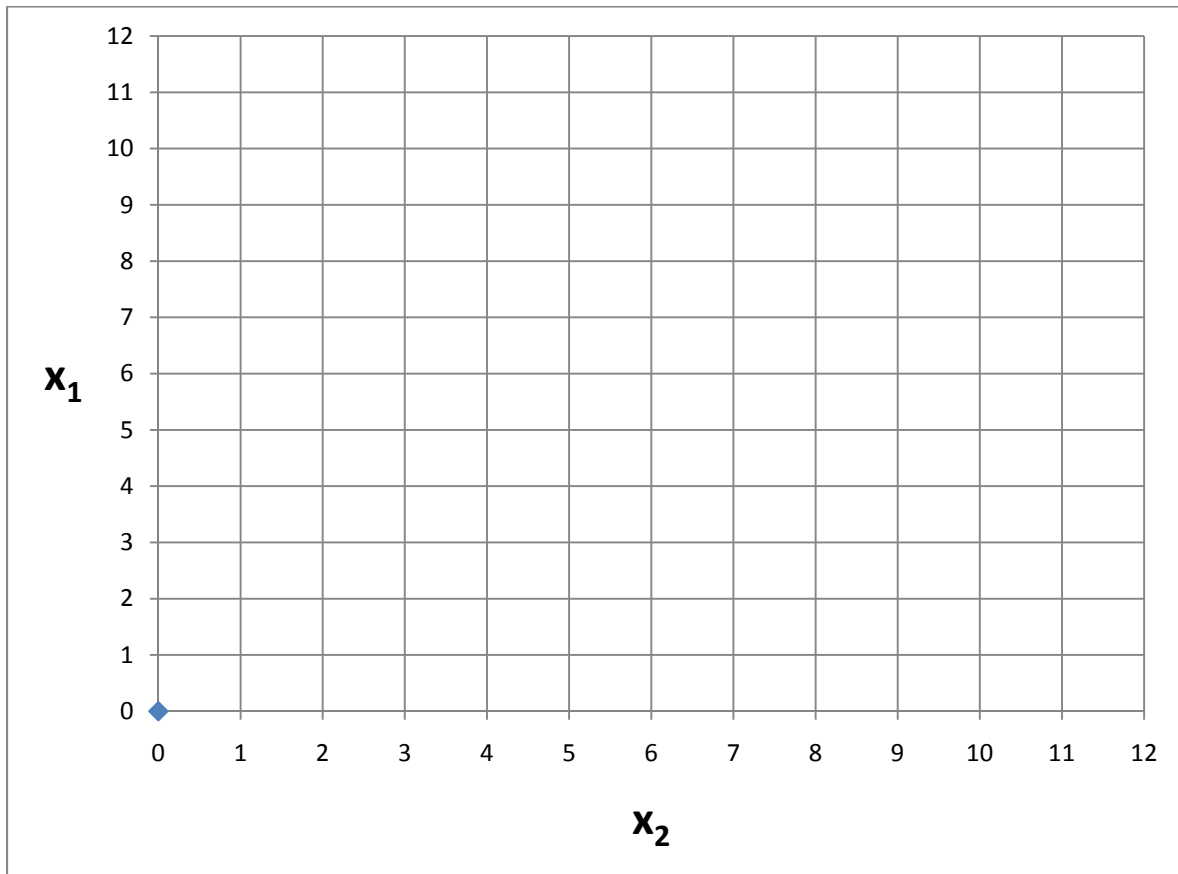
(4) [MRS: 8 pts] Suppose $y = f(x_1, x_2) = -2/x_1 - 4/x_2$. The arguments x_1 and x_2 are strictly positive.

- a. Find an expression for the marginal rate of substitution of x_2 for x_1 (that is, the formula for the |slope| of the level curves of y , with x_1 on the vertical axis and x_2 on the horizontal axis).

- b. Give an example of a different function $y = g(x_1, x_2)$ that has exactly the same formula for the marginal rate of substitution as $f(x_1, x_2)$.

IV. CRITICAL THINKING: [5 pts]

(1) Suppose $y = f(x_1, x_2)$. Further suppose $\partial y / \partial x_1$ is always exactly equal to $\partial y / \partial x_2$. Sketch two level curves of this function in the graph below.



[end of exam]