

**EXAMINATION #1 VERSION B**  
**“Mathematical Tools”**  
**September 7, 2011**

**INSTRUCTIONS:** This exam is closed-book, closed-notes. Calculators, mobile phones, and wireless devices are NOT permitted. Point values for each question are noted in brackets. As usual in this course, “ $exp(x)$ ” denotes the exponential function (also written  $e^x$ ) while “ $ln(x)$ ” denotes the natural logarithm function (logarithm to base  $e$ ).

**I. MULTIPLE CHOICE:** Circle the one best answer to each question. Use margins for scratch work. [3 pts each—33 pts total]

(1) Suppose  $y = 2x^2 + 5x + 9$ . Then the derivative of  $y$  with respect to  $x$  is

- a.  $dy/dx = 2$ .
- b.  $dy/dx = 4$ .
- c.  $dy/dx = 2x + 5$ .
- d.  $dy/dx = 5x + 9$ .
- e.  $dy/dx = 4x + 5$ .
- f.  $dy/dx = 2x^2 + 5x + 9$ .

(2) Suppose  $y = 2(7x+3)^{-4}$ . Then the derivative of  $y$  with respect to  $x$  is

- a.  $dy/dx = 2$ .
- b.  $dy/dx = -8$ .
- c.  $dy/dx = 2(7x+3)^{-5}$ .
- d.  $dy/dx = (7x+3)^{-5}$ .
- e.  $dy/dx = -8(7x+3)^{-5}$ .
- f.  $dy/dx = -56(7x+3)^{-5}$ .

(3) Which of the following functions has constant slope (or derivative)?

- a.  $y = 2 \ln(5x)$ .
- b.  $y = 2 \exp(5x)$ .
- c.  $y = 2 + 5x$ .
- d.  $y = 2 + (5/x)$ .
- e.  $y = 2 + 5x + 9x^2$ .
- f.  $y = 2x^{-5}$ .

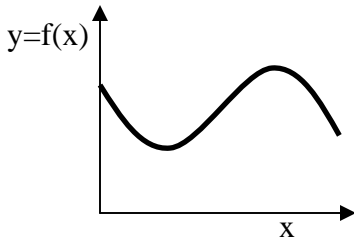
(4) If  $x$  increases by 3 percent, then  $\ln(x)$  increases by about

- a. 0.03 percent.
- b. 3 percent.
- c. 3 units.
- d. 0.03 units.
- e.  $\ln(3)$ , or about 1.1 units.

(5) Suppose we wish to maximize the function  $y = f(x)$ , which is continuously differentiable. Assuming there are no restrictions on the possible values of  $x$ , the maximizing value  $x^*$  must satisfy

- a.  $dy/dx = 0$ , if  $x = x^*$ .
- b.  $f(x^*) = 0$ .
- c.  $x^* = 0$ .
- d.  $d^2y/dx^2 = 0$ , if  $x = x^*$ .
- e. All of the above.

The next question refers to the following graph of  $y = f(x)$ .

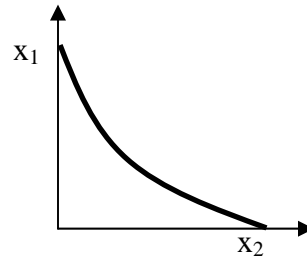


- (6) In this graph, the derivative of  $y$  with respect to  $x$  (that is,  $df/dx$ ) equals zero at
- no point on the graph.
  - one point on the graph.
  - two points on the graph.
  - three points on the graph.
  - four points on the graph.
  - more than four points on the graph.

- (7) Suppose  $y$  and  $x$  are strictly positive variables. If the derivative ( $dy/dx$ ) is *positive*, then the elasticity of  $y$  with respect to  $x$
- must be positive.
  - must be negative.
  - must be zero.
  - can be positive or negative but not zero.
  - can be positive, negative, or zero.

- (8) Which of the following functions has constant elasticity?
- $y = 2 \ln(5x)$ .
  - $y = 2 \exp(5x)$ .
  - $y = 2 + 5x$ .
  - $y = 2 + (5/x)$ .
  - $y = 2 + 5x + 9x^2$ .
  - $y = 2x^{-5}$ .

The next two questions refer to the following graph of a level curve, or contour, of the function  $y = f(x_1, x_2)$ .



- (9) As we move along this curve down and to the right, which is constant?
- $y$ .
  - $x_2$ .
  - both  $x_1$ , and  $x_2$ .
  - the marginal rate of substitution.
  - all of the above.

- (10) According to this graph, if  $x_2$  increases and  $y$  is to be held constant, then  $x_1$  must
- equal zero.
  - increase.
  - decrease.
  - remain constant.
  - cannot be determined from the information given.

- (11) Which of the following functions has constant partial derivatives ( $\partial y/\partial x_1$  and  $\partial y/\partial x_2$ )?
- $y = 8x_1^3 x_2^2$ .
  - $y = 8(x_1-3)^2 (x_2-2)^2$ .
  - $y = 8 + 3x_1 + 2x_2$ .
  - $y = 8x_1 + 3x_2 + 2(x_1x_2)^{1/2}$ .
  - $y = 8 + 3x_1^{-1} + 2x_2^{-1}$ .
  - $y = 8 + 3x_1^{1/2} + 2x_2^{1/2}$ .

**II. SHORT ANSWER:** Please write your answers in the boxes on this question sheet. Use margins for scratch work.

(1) [8 pts] Suppose the derivative of the function  $y = f(x)$  equals  $-4$  at a particular value of  $x$ . Moreover, the elasticity of  $y$  with respect to  $x$  equals  $-0.6$ .

First, suppose that  $x$  increases by 0.5 units.

a. Will  $y$  *increase* or *decrease*?

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b. By about how much?

units
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Alternatively, suppose that  $x$  decreases by 5 percent.

c. Will  $y$  *increase* or *decrease*?

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d. By about how much?

%
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(2) [8 pts] Consider the function  $y = f(x_1, x_2)$ . Suppose at a particular point,  $\partial y / \partial x_1 = 2$ , and  $\partial y / \partial x_2 = 4$ , and that the partial elasticities are  $\epsilon_1 = 1$  and  $\epsilon_2 = 2.5$ .

First suppose that  $x_1$  decreases by 0.4 and simultaneously  $x_2$  increases by 0.5.

a. Will  $y$  *increase* or *decrease*?

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b. By about how much?

units
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Alternatively, suppose that  $x_1$  increases by 4 percent and  $x_2$  decreases by 2 percent.

c. Will  $y$  *increase* or *decrease*?

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d. By about how much?

%
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(3) [6 pts] Consider the function  $y = f(x_1, x_2)$ . Suppose at a particular point,  $\partial y / \partial x_1 = 3$ , and  $\partial y / \partial x_2 = 6$ .

a. Does the level curve of the function slope *up* or *down* at that point?

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Further suppose  $x_2$  increases by 1 unit, but suppose we want to keep the value of  $y$  constant.

b. Must  $x_1$  *increase* or *decrease*?

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c. By about how much?

units
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(4) [4 pts] Revenue equals price times quantity sold. Suppose price decreases by 3 percent and the quantity sold increases by 4 percent.

a. Will revenue *increase* or *decrease*?

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b. By about how much?

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(5) [4 pts] Average cost equals total cost divided by total output. Suppose total output decreases by 2 percent and total cost decreases by 6 percent.

a. Will the average cost *increase* or *decrease*?

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b. By about how much?

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**III. PROBLEMS:** Please write your answers in the boxes on this question sheet. Show your work and circle your final answers.

(1) [Optimization: 8 pts] Consider the function  $y = f(x) = 9x^{1/2} - (x/2)$ .

a. Find a formula (in terms of  $x$ ) for the derivative of  $y$  with respect to  $x$  ( $dy/dx$ ).

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b. Compute the value  $x^*$  that maximizes this function.

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(2) [Partial elasticities: 8 pts] Suppose  $y = x_1^3 (x_2 - 4)^2$ .

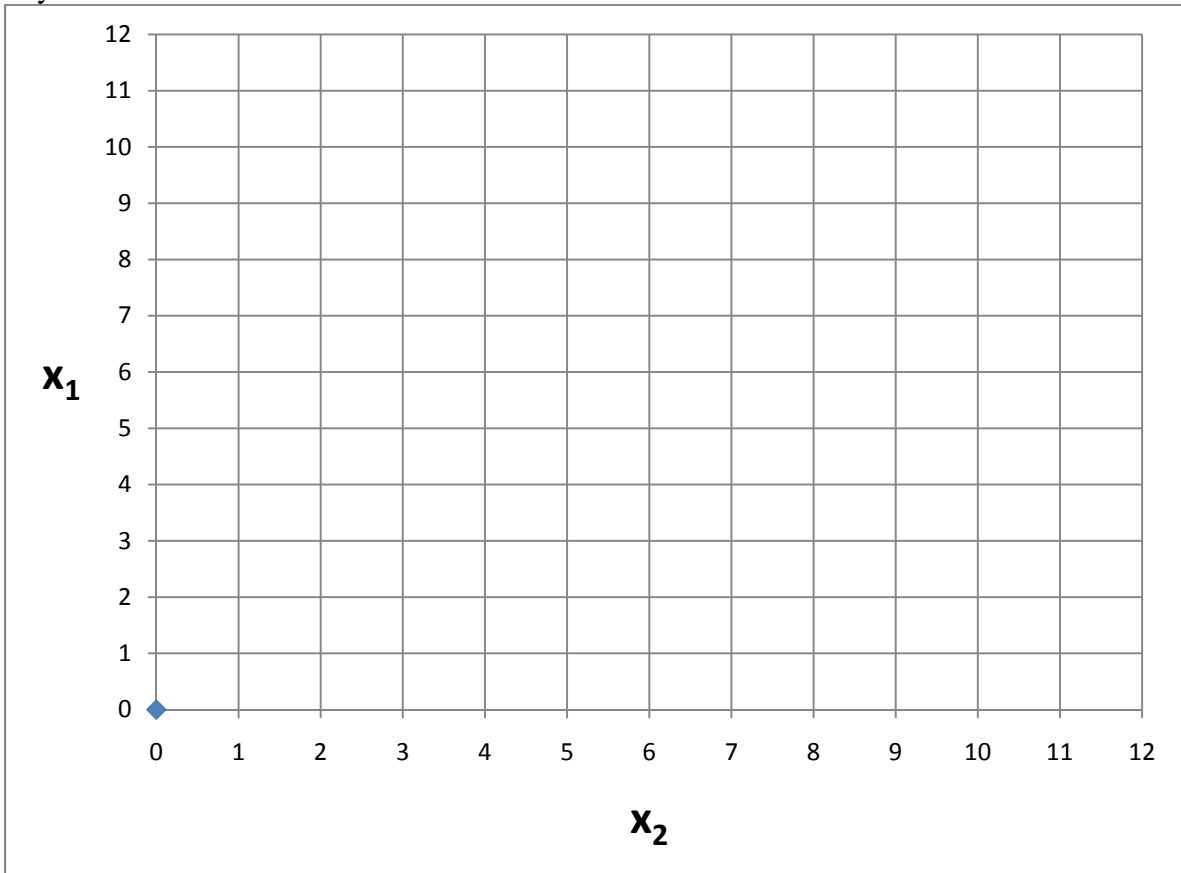
- a. Find a formula for  $\varepsilon_1$ , the partial elasticity of  $y$  with respect to  $x_1$ . Express your answer in terms of  $x_1$  and  $x_2$  alone, not  $y$ .

- b. Find a formula for  $\varepsilon_2$ , the partial elasticity of  $y$  with respect to  $x_2$ . Express your answer in terms of  $x_1$  and  $x_2$  alone, not  $y$ .

(3) [MRS: 8 pts] Consider the function  $y = 0.5 x_1 + 0.25 x_2$ .

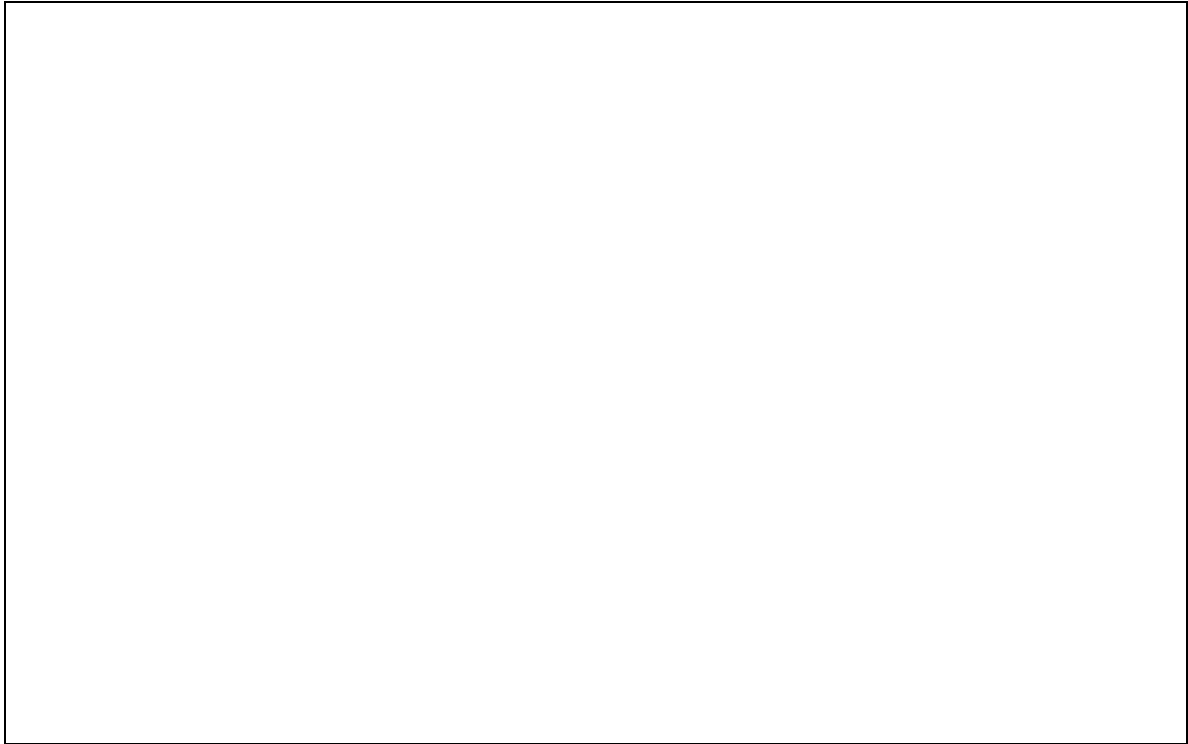
- a. Give the marginal rate of substitution of  $x_2$  for  $x_1$  (that is, is the |slope| of the level curve with  $x_1$  on the vertical axis and  $x_2$  on the horizontal axis) for this function.

- b. Carefully draw the level curve of this function when  $y = 1$  and the level curve when  $y = 3$ .

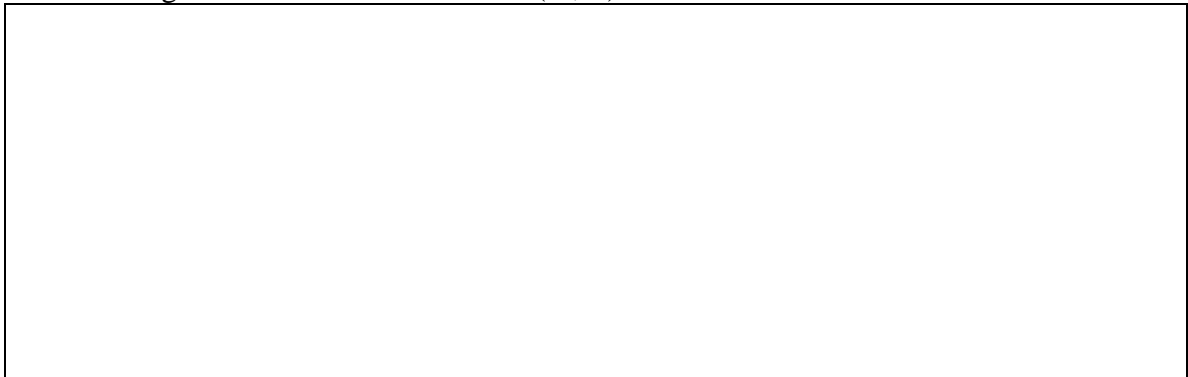


(4) [MRS: 8 pts] Suppose  $y = f(x_1, x_2) = 4x_1^{1/2} + 6x_2^{1/2}$ . The arguments  $x_1$  and  $x_2$  are strictly positive.

- a. Find an expression for the marginal rate of substitution of  $x_2$  for  $x_1$  (that is, the formula for the |slope| of the level curves of  $y$ , with  $x_1$  on the vertical axis and  $x_2$  on the horizontal axis).

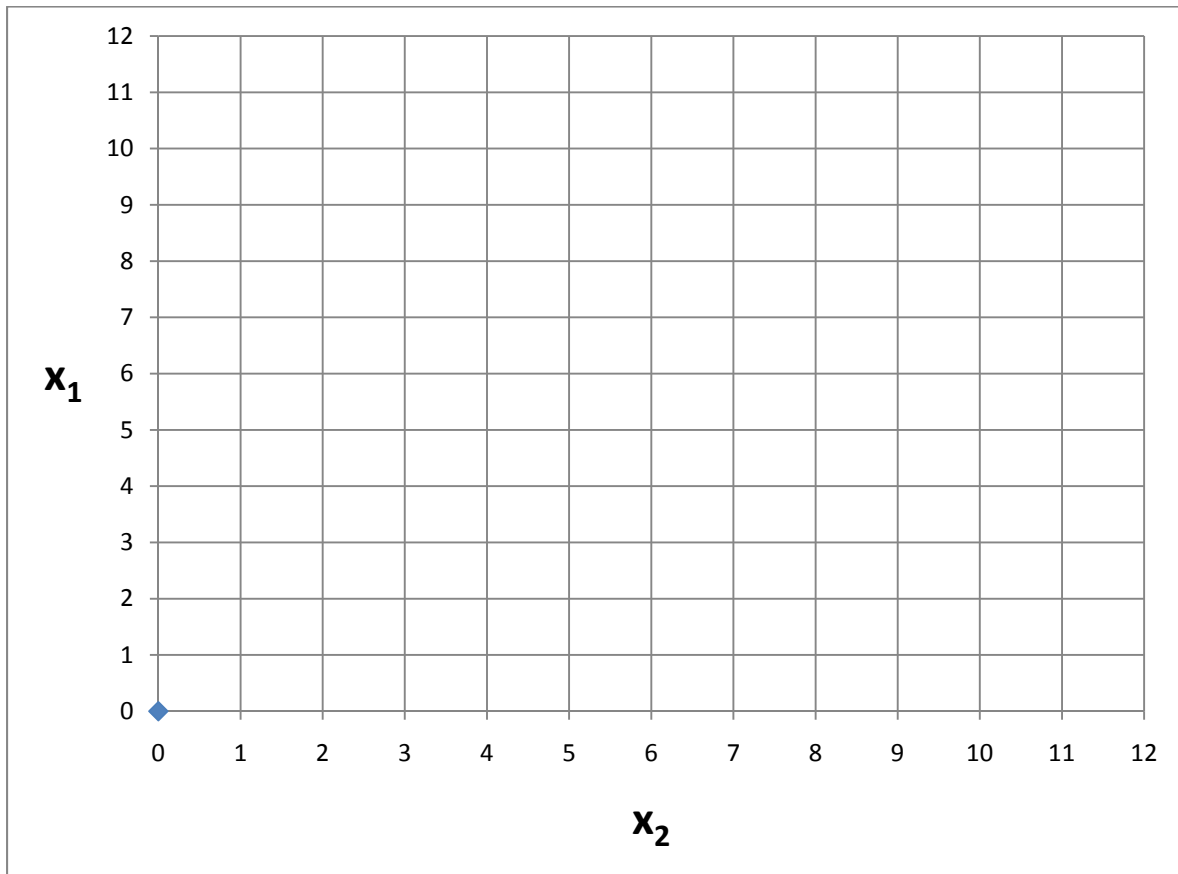


- b. Give an example of a different function  $y = g(x_1, x_2)$  that has exactly the same formula for the marginal rate of substitution as  $f(x_1, x_2)$ .



**IV. CRITICAL THINKING:** [5 pts]

(1) Suppose  $y = f(x_1, x_2)$ . Further suppose  $\partial y / \partial x_1$  is always equal to zero, but  $\partial y / \partial x_2$  nonzero. Sketch two level curves of this function in the graph below.



[end of exam]